

ADDENDUM TO A TEST SUITE FOR THE CALCULATION OF  
TIME DOMAIN FREQUENCY STABILITY

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**ABSTRACT**

This paper is an addendum to the time domain frequency stability test suite presented at the 1995 IEEE Frequency Control Symposium. It adds values for the new TOTALDEV statistic, and for the recently re-emphasized Hadamard deviation. Values are given for those quantities at several averaging factors for both the classic NBS data set and the 1000-point portable test suite. Plots are also presented that show the improved performance of the TOTALDEV statistic as compared with the normal Allan variance for averaging times that are a significant fraction of the record length. Also included is a correction to the value given in the original paper for the modified Allan deviation for the 1000-point test suite with an averaging factor of 100.

**INTRODUCTION**

The need for a test suite to validate software for the calculation of time domain frequency stability was discussed in a previous paper [1], along with values for several common statistics for the classic NBS test data [2] and a 1000-point portable test suite. This addendum to that paper presents test values for two additional quantities, the new TOTALDEV statistic [3],[4] and the Hadamard deviation [5],[6]. The usefulness of the latter has recently been re-emphasized for the characterization of GPS rubidium clocks [7]. In addition, as an example of the significant improvement that is offered by TOTALDEV at averaging times that are a substantial fraction of the record length, plots are presented comparing ADEV and TOTALDEV for the 1000-point test suite.

**TOTALVAR**

The Total Allan variance, TOTALVAR, is a new version of the normal Allan variance, AVAR. TOTALVAR wraps the frequency data in a circular fashion [8] at every basic measurement interval  $\tau_0$ , and averages the re-

sulting AVAR values at averaging time  $\tau = m\tau_0$ . This improves significantly the estimate of frequency stability, especially for long averaging times. As an example, Figures 1 and 2 compare ADEV and TOTALDEV for the 1000-point test suite at every averaging time out to one-fifth of the record length. The large fluctuations present in the ADEV values are largely eliminated in TOTALDEV.

Test values for TOTALDEV are given in Tables I and II for the NBS and 1000-point data sets respectively.

**HADAMARD VARIANCE**

The Hadamard variance is a 3-sample variance that is similar to the 2-sample Allan variance. It examines the 2nd difference of the fractional frequencies, the 3rd difference of the phase variations. Because of this, the Hadamard variance converges for  $\alpha > -5$  power-law noise types. It is also unaffected by linear frequency drift. Like the Allan variance, it is usually expressed as its square-root, the Hadamard deviation.

Test values for the Hadamard deviation are given in Tables I and II for the NBS and 1000-point data sets respectively.

**CORRECTION**

The value of the modified Allan deviation for the 1000-point test suite with an averaging factor of 100 was incorrect in the original paper (it repeated the value for the normal Allan deviation). The corrected value (2.170921e-02) is also shown in Table II.

**REFERENCES**

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6. T. Walter, "A Multi-Variance Analysis in the Time Domain", *Proc. 1992 PTTI Meeting*, pp. 413-436, Dec. 1992.
7. S.T. Hutsell, "Relating the Hadamard Variance to MCS Kalman Filter Clock Estimation," *Proc. 1995 PTTI Meeting*, Dec. 1995 (to be published).
8. D.A. Howe, "Circular Representation of Infinitely Extended Sequences," *Proc. 1995 IEEE Freq. Contrl. Symp.*, pp. 337-345, June 1995.

Table I. NBS Monograph 140, Annex 8.E Test Data Statistics		
Averaging Factor	1	2
# Data Points	9	4
Normal Allan Deviation	91.22945	115.8082
Total Allan Deviation	99.83041	76.40517
Hadamard Deviation	70.80608	116.7980

Table II. 1000-Point Test Suite Frequency Data Set Statistics			
Averaging Factor	1	10	100
# Data Points	1000	100	10
Normal Allan Deviation	2.922319e-01	9.965736e-02	3.897804e-02
Total Allan Deviation	2.921054e-01	9.111929e-02	3.250070e-02
Hadamard Deviation	2.943883e-01	1.052754e-01	3.910860e-02
Modified Allan Deviation	2.922319e-01	6.172376e-02	2.170921e-02

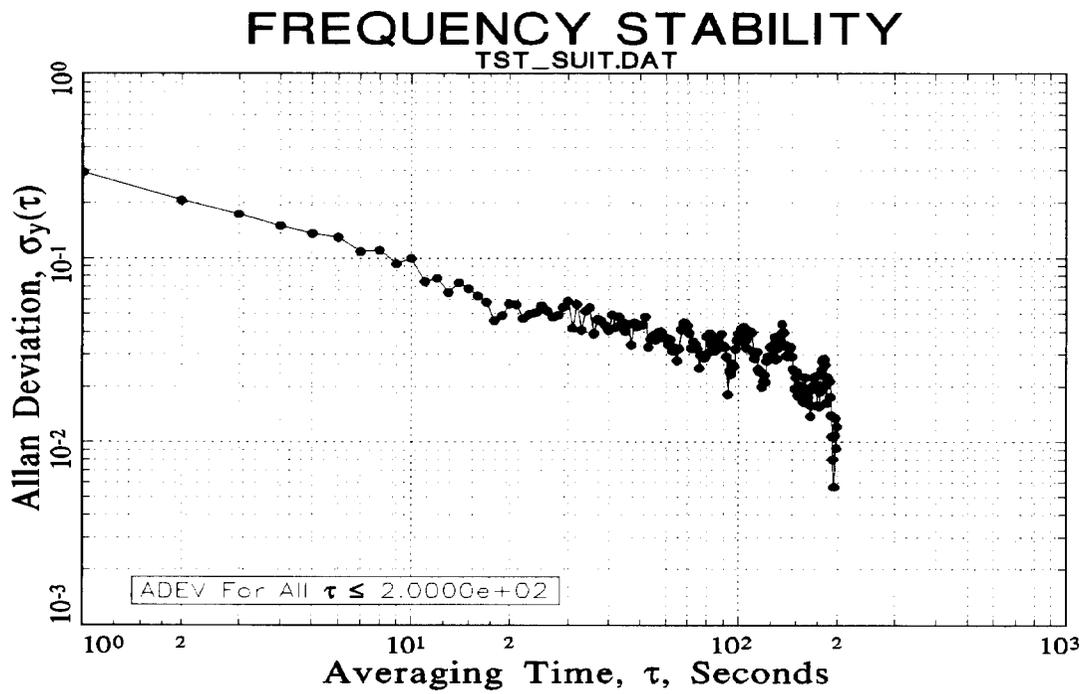


Figure 1: ADEV Plot for Test Suite Data

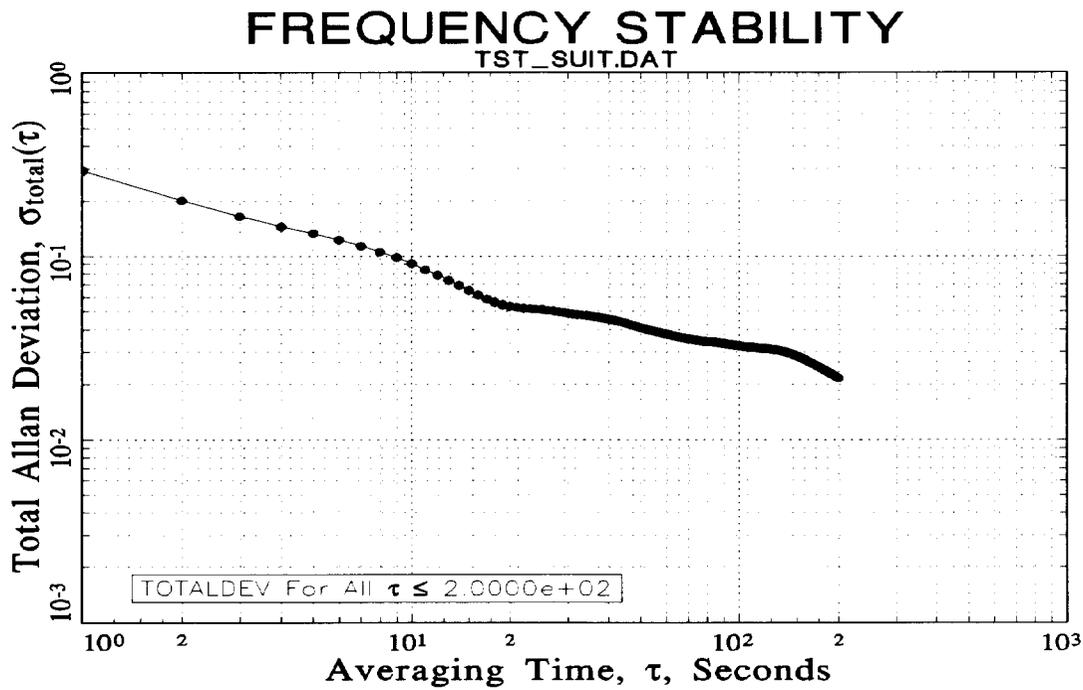


Figure 2: TOTALDEV Plot for Test Suite Data