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Single Sideband Modulation using Sequence Asymmetric Polyphase Networks

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Single Sideband Modulation using Sequence Asymmetric Polyphase Networks

A sequence asymmetric filter can split an input signal into components which, when applied to a polyphase modulator, generate a single sideband signal. This method of single sideband generation is much less sensitive to filter component tolerances than is quadrature modulation.

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Introduction

The requirement for economic and efficient single sideband modulation has attracted considerable research and development over the last 50 years. The first single sideband modulators used direct amplitude modulation which produced both upper and lower sidebands, one of which was selected by a band pass filter (Figure 1). This is still the most popular method today although considerable effort has been spent in the search for alternatives. Many systems impose such stringent requirements on the suppression of the unwanted sideband that band pass filters become uneconomic.

One of the best known alternatives is quadrature modulation (Figure 2) in which the signal to be modulated is split into two paths. A phase shift network arranges the signals in these two paths so that they are approximately 90 degrees out of phase over a defined bandwidth. Applying each signal to its respective modulator, with the carriers 90 degrees apart, and then adding the resultant modulation products gives controlled suppression of one sideband. There are a number of variants, but they all suffer from the disadvantage of high sensitivity to the tolerances of the components in the phase shift networks.

There is also a third method, the *N*-path filter method, due to Barber, Weaver, and others, which relies on double modulation and filtering (Figure 3). The principle is similar to quadrature modulation except that the initial 90-degree phase splitting of the signal is achieved using an additional set of modulators. In practical arrangements, the effective filtering characteristic is symmetric about the center carrier frequencies — although the virtual carrier is at the edge of the wanted sideband as usual. Because the modulators operate with carrier frequencies in the center of the transmission band, any carrier leak (caused by modulator imbalance, for example) appears as a steady tone at a frequency where it is most objectionable. Implementation of a high quality transmission system using this method therefore involves some practical difficulties.

The equations given with Figure 3 show that the low pass filter characteristic is shifted by the modulation frequency. The asymmetric nature of the sideband suppression requirements in frequency division multiplex systems and the problems associated with the *N*-path method make it desirable to have a filter network that overcomes these difficulties. Such networks can be constructed and because of their properties they are known as sequence asymmetric polyphase filters. Also, when such networks are used it is possible to devise a much simpler single sideband modulator that dispenses with the input modulators of the *N*-path modulator while retaining many of the advantages over quadrature modulation.

Some of these filters, suitable for single sideband modulation, can be constructed using only resistors and capacitors, making thick or thin film construction a practical possibility.

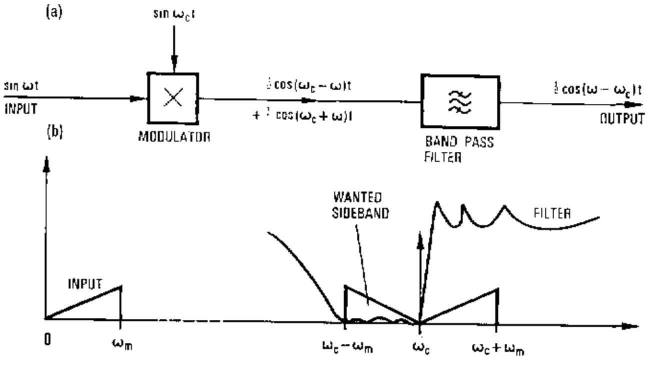


Figure 1 - Conventional single sideband modulation.

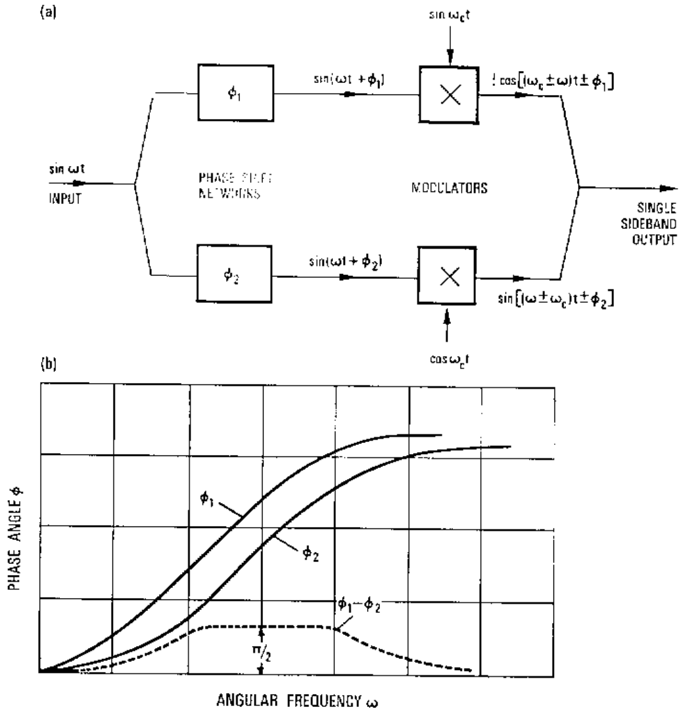
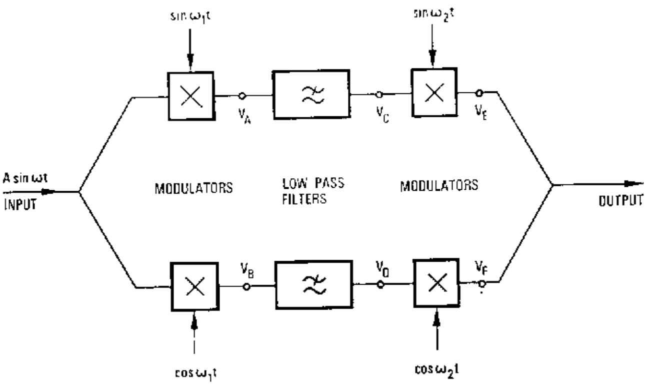


Figure 2 - Quadrature modulation.
(a) Modulation procedure.
(b) Characteristic of all pass (phase shift) networks.

Sequence Asymmetric Polyphase Filters

Definition of a Polyphase Network

For the purpose of this article a polyphase network is a network which has N input terminals and N output terminals, where there is no limit to the value of N. In practice, however, N is generally limited to 2, 3, or 4 for economic reasons. The networks are physically symmetric: that is, the electrical paths from each input terminal to its corresponding output terminal are identical, as shown in Figure 4.



INPUT – $A \sin \omega t$

$$V_A = \frac{A}{2}\left[\cos(\omega - \omega_1)t - \cos(\omega + \omega_1)t\right]$$
$$V_B = \frac{A}{2}\left[\sin(\omega - \omega_1)t + \sin(\omega + \omega_1)t\right]$$
$$V_C = \frac{A}{2}\cos(\omega - \omega_1)t \; \left|H(p - p_1)\right|$$
$$V_D = \frac{A}{2}\sin(\omega - \omega_1)t \; \left|H(p - p_1)\right|$$
$$V_E = \frac{A}{4}\left[\sin(\omega + \omega_2 - \omega_1)t + \sin(\omega_1 + \omega_2 - \omega)t\right] \; \left|H(p - p_1)\right|$$
$$V_F = \frac{A}{4}\left[\sin(\omega + \omega_2 - \omega_1)t - \sin(\omega_1 + \omega_2 - \omega)t\right] \; \left|H(p - p_1)\right|$$
$$\text{OUTPUT} = V_E + V_F = \frac{A}{2}\sin(\omega + \omega_2 - \omega_1) \; \left|H(p - p_1)\right|$$

Figure 3 - N-path filter method of single sideband modulation.

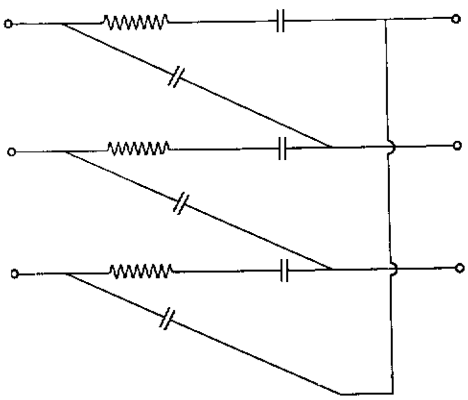


Figure 4 - Symmetric polyphase network. The electrical paths from each input terminal to its corresponding output terminal are identical.

Polyphase Signals, Symmetric Components, and Negative Frequency

A polyphase signal is a set of N vectors, each of which corresponds to a voltage that might be applied to one of the N input terminals of the polyphase filter. In a symmetric polyphase signal the vectors are of equal magnitude and are spaced equally in phase.

According to the theory of symmetric components, any unbalanced system of N vectors can be represented as the sum of N symmetric vector systems. As an example, Figure 5 shows the resolution of an asymmetric 4-phase system of vectors into the sum of 4 symmetric sets of 4-phase vectors, giving the following set of equations:

$$a_1' = a_0 - ja_1 - a_2 + ja_3$$
$$a_2' = a_0 - a_1 + a_2 - a_3$$
$$a_3' = a_0 + ja_1 - a_2 - ja_3$$
$$a_4' = a_0 + a_1 + a_2 + a_3.$$

Inverting the matrix of coefficients gives:

$$a_0 = \frac{1}{4}(a_1' + a_2' + a_3' + a_4')$$
$$a_1 = \frac{1}{4}(ja_1' - a_2' - ja_3' + a_4')$$
$$a_2 = \frac{1}{4}(-a_1' + a_2' - a_3' + a_4')$$
$$a_3 = \frac{1}{4}(-ja_1' - a_2' + ja_3' + a_4')$$

where a_1', a_2', a_3', and a_4' represent the asymmetric vectors and a_0, a_1, a_2, and a_3 represent 1 of the 4 vectors in each of the 4 respective symmetric systems.

In Figure 5, sequence 1 may be considered of positive value because phase 1 leads phase 2 by 90 degrees. However, in sequence 3 phase 1 lags phase 2 and this is therefore considered to be of negative value. If the vectors were to rotate the opposite way, sequence 1 would become negative and sequence 3 positive. Alternatively, they could both be considered to be the same in sequence but with one having a negative frequency.

Sequence Asymmetric Property

Although sequence asymmetric polyphase networks are generally physically symmetric, they exhibit dif-

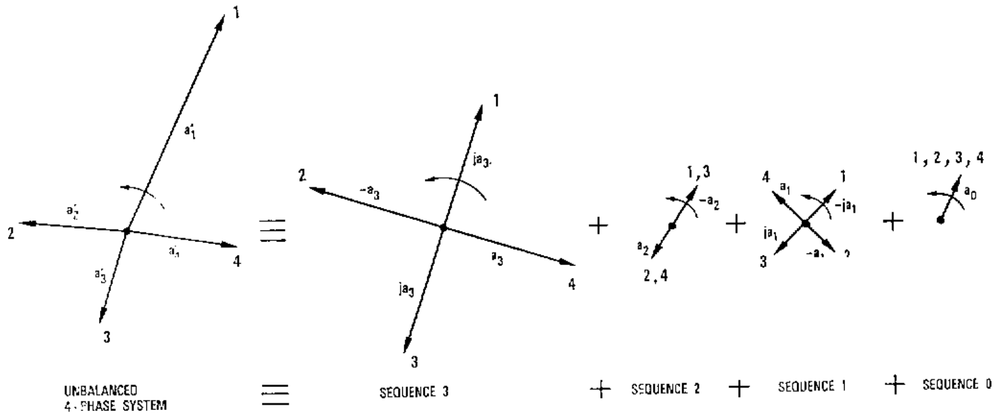


Figure 5 - Resolution of asymmetric system of vectors into 4-phase symmetric components. The numbers at the end of each vector refer to the paths in which the vectors are observed.

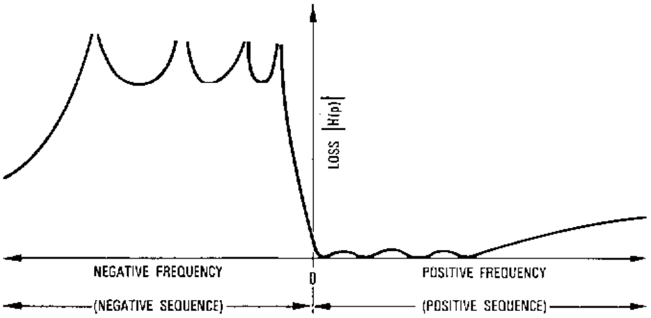


Figure 6 - Different insertion loss characteristics exhibited by a sequence asymmetric filter for positive and negative sequence inputs.

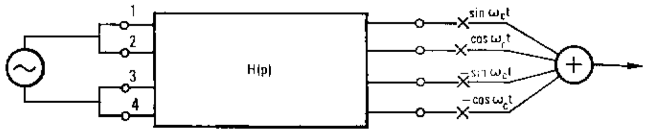


Figure 7 - Sequence asymmetric filter used in a single sideband modulator.

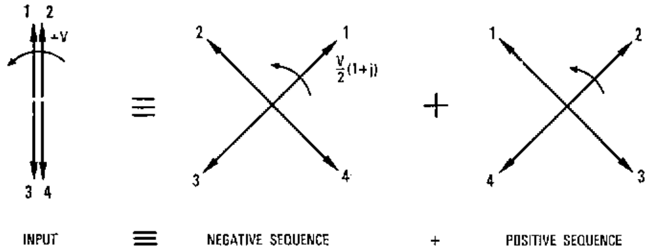


Figure 8 - Resolution of modulator input into symmetric components.

ferent insertion loss characteristics for inputs of opposite sequence, as shown in Figure 6.

Use in Single Sideband Modulation

Consider a 4-phase network driven from a single balanced input connected as shown in Figure 7. Using the theory of symmetric components, the input signal can be split into two 4-phase symmetric components of opposite sequence as shown in Figure 8. The action of the following network can then be analyzed and the final value at the output obtained by superposition.

If the response of the polyphase filter to the positive sequence input is $H(p)$, then it will be $H(-p)$ to the negative sequence input.

By superposition the following signals exist at the 4 output terminals:

$$V_1 = \frac{V}{2}(1+j)\,[jH(p) + H(-p)]$$

$$V_2 = \frac{V}{2}(1+j)\,[H(p) + jH(-p)]$$

$$V_3 = \frac{V}{2}(1+j)\,[-jH(p) - H(-p)]$$

$$V_4 = \frac{V}{2}(1+j)\,[-H(p) - jH(-p)]$$

where V is the input voltage and $p = j\omega$.

If each of the outputs is connected to a modulator with carriers of $\sin\omega_c t$, $\cos\omega_c t$, $-\sin\omega_c t$, and $-\cos\omega_c t$, respectively, and the resultant modulation products are summed as shown in Figure 7, then the output voltage V_0 is:

$$\begin{aligned}V_0 = {} & \frac{1}{4j}(1+j)\,[jH(p)+H(-p)] \times \\ & \times [V(p-p_c) - V(p+p_c)] \\ & + \frac{1}{4}(1+j)\,[H(p)+jH(-p)] \times \\ & \times [V(p-p_c) + V(p+p_c)] \\ & - \frac{1}{4j}(1+j)\,[-jH(p)-H(-p)] \times \\ & \times [V(p-p_c) - V(p+p_c)] \\ & - \frac{1}{4}(1+j)\,[-H(p)-jH(-p)] \times \\ & \times [V(p-p_c) + V(p+p_c)]\end{aligned}$$

which reduces to

$$V_0 = (1+j)\,H(p)\,V(p-p_c) - (1-j)\,H(-p)\,V(p+p_c).$$

The first term represents the lower sideband attenuated by $H(p)$ and the second represents the upper sideband attenuated by $H(-p)$. The effect is therefore similar to conventional single sideband modulation where the signal is passed through a modulator followed by a sideband selection filter with a transfer function $H(p+p_c)$. The basic method can be used for any number of phases and with different network drive arrangements. The circuit can also be used in reverse for demodulation. Practical analog modulators are most easily constructed using switches that introduce modulation products at harmonics of the carrier frequency. It can be shown that with an N-phase modulator all unwanted products up to and including the $(N-2)$th harmonic are cancelled. There is, therefore, a good reason for making N as large as possible.

Practical Polyphase Networks

Passive RC Networks

To make a polyphase network sequence selective it is necessary to interconnect the phases so that the response can be modified depending on whether a particular signal leads or lags its neighbors.

Figure 9 shows a 4-phase network where the cross

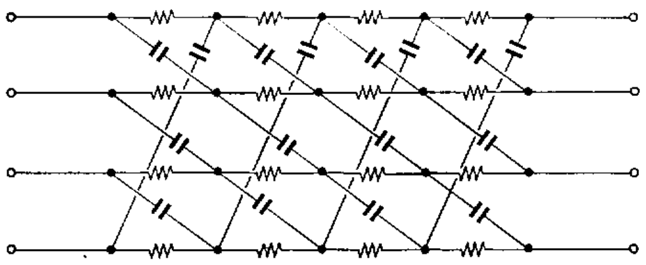


Figure 9 - A 4-section 4-phase RC polyphase network.

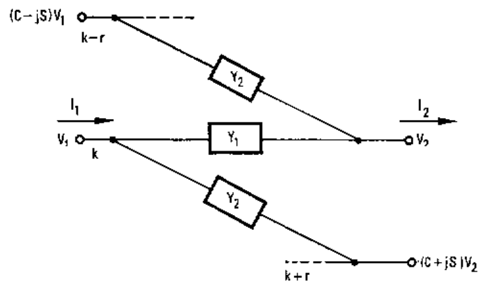


Figure 10 - One phase of a generalized N-phase filter section $C = \cos 2\pi r/N$ $S = \sin 2\pi r/N$.

connection uses skewed capacitors. The frequency response of this network is shown in Figure 6. Each section consists of 4 identical resistors and 4 identical capacitors and gives rise to one of the attenuation peaks of Figure 6. Equal minima stop bands and equal ripple pass bands can be realized. Because each phase is identical to its neighbors, it is possible to write the transfer function for a single phase as a chain matrix.

Figure 10 shows one phase of a generalized N-phase filter section where an admittance Y_1 is connected between the input and output terminals of the same path, and an admittance Y_2 is skewed between input and output terminals across r paths. In this case the chain matrix can be written:

$$\begin{bmatrix} V_1 \\ \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{Y_1+Y_2}{Y_1+CY_2-jSY_2} & \dfrac{1}{Y_1+CY_2-jSY_2} \\ \dfrac{2(1-C)\,Y_1Y_2}{Y_1+CY_2-jSY_2} & \dfrac{Y_1+Y_2}{Y_1+CY_2-jSY_2} \end{bmatrix} \begin{bmatrix} V_2 \\ \\ I_2 \end{bmatrix}.$$

For the simple case of Figure 9 where $N=4$, $r=1$, $Y_1=1/R_1$, and $Y_2=pC_1$, the matrix becomes:

$$\begin{bmatrix} V_1 \\ \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{1+pC_1R_1}{1-jpC_1R_1} & \dfrac{R_1}{1-jpC_1R_1} \\ \dfrac{2pC_1}{1-jpC_1R_1} & \dfrac{1+pC_1R_1}{1-jpC_1R_1} \end{bmatrix} \begin{bmatrix} V_2 \\ \\ I_2 \end{bmatrix}.$$

This has zero gain at $\omega=-1/C_1R_1$ and maximum gain at approximately $\omega=1/C_1R_1$.

A wide variety of sections can be constructed on this basis, especially if the phases are interconnected in more complicated ways. However, analysis has shown that there is little advantage to be gained by departing from the simplest configurations where sections are connected in cascade, each contributing one attenuation peak.

Sensitivity of the transfer characteristic to component tolerances is much less than in the quadrature modulation counterpart. This can be deduced, without recourse to analysis, from the following considerations. When the network is used for single sideband generation, one sequence is almost completely attenuated. Starting from the input, which is 2-phase, the signal progresses down the network gradually becoming more and more perfect 4-phase as it proceeds. Each successive section improves the signal regardless of the preceding sections. Only the final sections need components with tolerances commensurate with the stop band requirements.

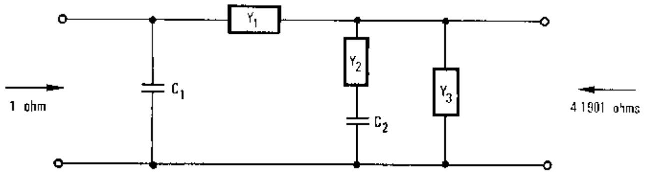


Figure 11 - Single phase asymmetric band pass filter.
$C_1 = 1{\cdot}328$ farads
$C_2 = 1{\cdot}5809$ farads
$Y_1 = -\,j\,0{\cdot}5359$ mho
$Y_2 = +\,j\,0{\cdot}5359$ mho
$Y_3 = +\,j\,0{\cdot}3476$ mho.

Even for these sections, the critical tolerances are ratios between, say, 4 identical resistors or 4 identical capacitors. Absolute tolerances result in a small shift in the position of the attenuation peak which is generally not so serious.

This contrasts with quadrature modulation in which the 90-degree phase splitting is achieved by taking the difference between two large phase shifts from two different networks. In consequence, sideband suppression is very sensitive to the tolerance of every component in both networks. Statistical analysis has shown that the passive RC polyphase modulator is an order of magnitude less sensitive to component tolerances than the RC network quadrature modulation counterpart.

Active Polyphase Networks

Passive polyphase networks somewhat restrict the shape of the transfer function that can be obtained because the attenuation zeros are limited to the negative real axis of the p plane. When more flexibility is required active networks must be used, as LC or RLC networks alone are not sufficient.

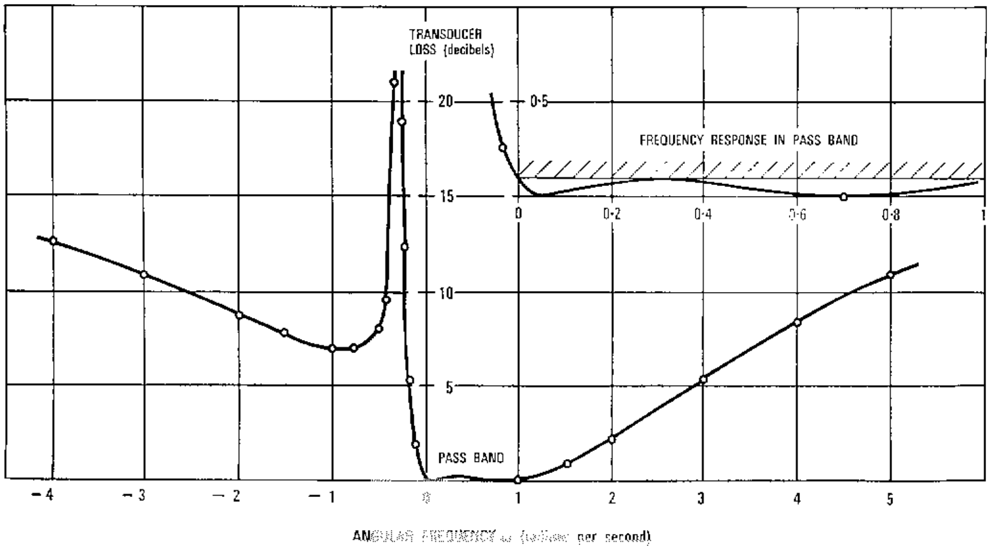


Figure 12 - Frequency response of the transducer loss for the filter of Figure 11.

If a transfer function is synthesized that is asymmetric about zero frequency it may contain imaginary terms in even powers of ω (frequency) and real terms in odd powers of ω. When a single phase network is synthesized from such a function it will contain elements of constant reactance (fixed value imaginary resistors). Such elements are not realizable in a single phase network but can be realized in polyphase form using active devices.

Figure 11 shows the single phase configuration of a network synthesized to be a simple asymmetric band pass filter. Figure 12 shows the frequency response.

Figure 13 shows one method of realizing constant reactances in 4-phase using controlled current sources in what is virtually a 4-port gyrator. Since all paths carry signals that are identical except in phase, it is possible to induce a current to flow in quadrature with the voltage drop thus establishing an effective constant reactance. One network (such as that in Figure 11) would be used per phase of the polyphase network together with an N-port gyrator per constant reactance.

Other methods are possible using devices such as $1:j$ impedance transformers, negative impedance convertors, and operational amplifiers

Practical Development

Polyphase Networks and Modulators

A number of circuits incorporating polyphase networks have been constructed and tested successfully. Among the more important are:

- Passive RC network modem giving 70 decibels single sideband suppression at 100 kilohertz and suitable for frequency division multiplex systems.
- Passive RC phase splitter giving 4-phase output accurate to 0·1 degree from 50 hertz to 20 kilohertz.
- Active polyphase filters in various forms and of up to 7th order (equivalent to 14th-order conventional channel band pass filter).
- Oscillators using polyphase networks in the feedback path.

One of the most important practical problems that has to be overcome is the design of satisfactory modulators. Series gate field effect transistors have proved to be the best as regards the critical parameters of phase accuracy, on-to-off impedance ratio, linearity, and carrier leak.

Network Synthesis and Analysis

Synthesis techniques have been developed for transfer functions that are asymmetric about zero frequency. Once the function is known, suitable networks can be synthesized using conventional techniques. Passive RC networks, however, present a more difficult problem. Low order types can be designed by analyzing the circuit and equating coefficients with the transfer function to determine the element values. However, the method becomes intractable for higher order filters and for these the best available method is computer optimization.

Monte Carlo tolerance analysis of RC polyphase networks has provided some interesting results. It has shown, for example, that a sideband suppression of 60 decibels can be achieved consistently in a network where the relative tolerances are graded from ± 2·5 percent at the input to ± 0·2 percent at the output.

Conclusions

Sequence asymmetric polyphase filters can be used in conjunction with a new and simpler single sideband modulator to provide efficient single sideband modulation. Polyphase networks can be constructed using passive or active components. One such network using only resistors and capacitors is particularly suited to thick or thin film construction.

The sequence selective polyphase filter can be seen as the link that unifies all the different known methods of single sideband modulation. The most general case would be an N-path modulator with the low pass filters replaced by a sequence asymmetric polyphase network. N-path and polyphase modulation are then special cases. Quadrature modulation can be considered as a special case of polyphase modulation in which the sequence selective polyphase network has been transformed into two separate phase splitting circuits with greatly increased sensitivity to component tolerances.

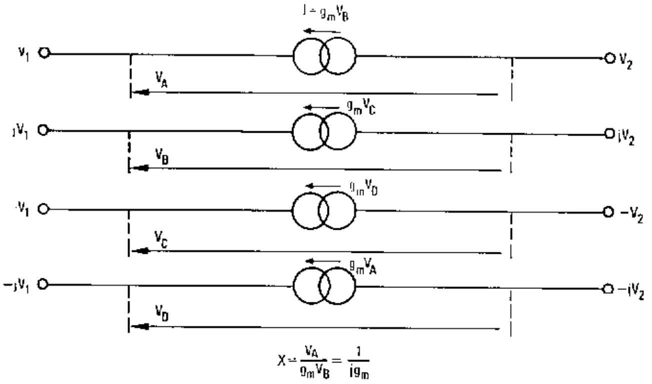


Figure 13 - Method of realizing a constant reactance using a polyphase gyrator.

Possible applications include single sideband radio, frequency division multiplex, vestigial sideband transmission, program quality transmission, phase measuring equipment, and frequency selective test gear.

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