CHAPTER 5

Applications

Sequence asymmetric filters can be applied in a wide range of systems where modulation or filtering is required. The principal modes of application to be discussed are in N path filtering and Single Sideband Modulation.

Phase splitting or combining, for which sequence asymmetric polyphase filters are eminently suitable, is also used in many cases without modulation. One example is in directional microphones where the outputs from a number of transducers are combined in a phase shifting network to produce a desired polar response pattern.

Miscellaneous applications include polyphase oscillators, frequency multiplication and pulse generation.

5.1 Application to N-Path Filters and Modulators

In 1947 N.F. Barber published a paper (Ref. D1) on band pass filtering using modulation. This was the vanguard of a large number of papers which expanded the subject and gave rise to the name 'N-path filter'. In 1956 D.K. Weaver, making a modification on the input modulators, published a paper (Ref. D5) on 'A third method of single sideband generation and detection'. This was followed in 1960 (Ref. D6) by the first comprehensive analysis of the generalized N path filter. With a few exceptions (B6, B7, D12) most publications to date have considered the N paths to be discrete and to contain individual networks. The exceptions consider additional networks connected between the paths.

This section will consider the application of sequence asymmetric polyphase filters in place of N individual filters.

5.1.1 Principles of N-Path Systems

As a basis for further development an analysis of N path filters and modulators will be given. The first part of the analysis was the joint work of D.R. Barber and the Author and was published in Ref. D11. The analysis is also similar to that of Franks and Sandberg (Ref. D6) who considered only N path filtering.

Figure 5.1.1 shows the structure of a basic N-path filter or modulator. It contains N identical paths each of which consists of a first modulator, a filter and a second modulator. The input signal after band-limiting is applied to the inputs of all N paths. The outputs of all N paths are summed and the summed output is passed through a band-limiting filter. Band-limiting is usually necessary because of the unwanted products produced by the practical modulation functions used.

Figure 5.1.2 shows one path in more detail and where in the time domain:

$V_1(t)$ is the input signal
$r(t)$ is the first modulating function
$h(t)$ is the filter transfer function
$q(t)$ is the second modulating function.

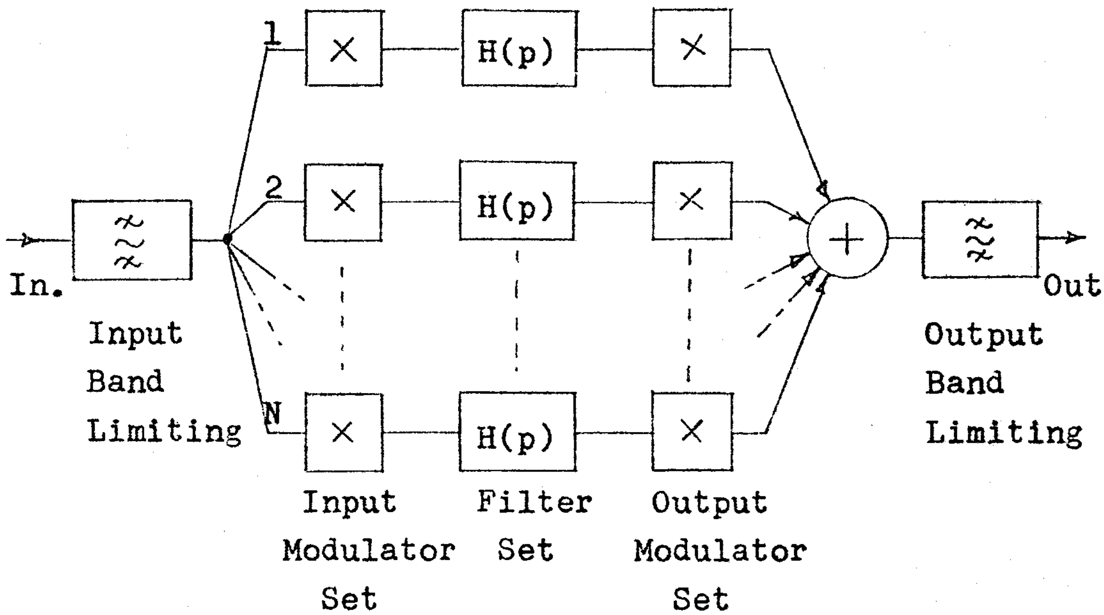


Figure 5.1.1 Basic N-path Filter/Modulator.

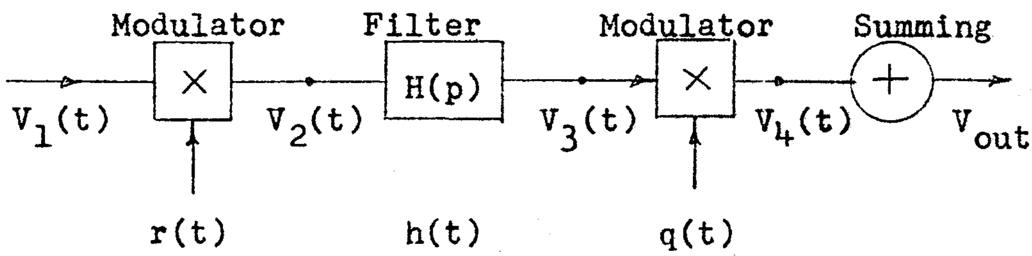


Figure 5.1.2 One path of the N-path Filter

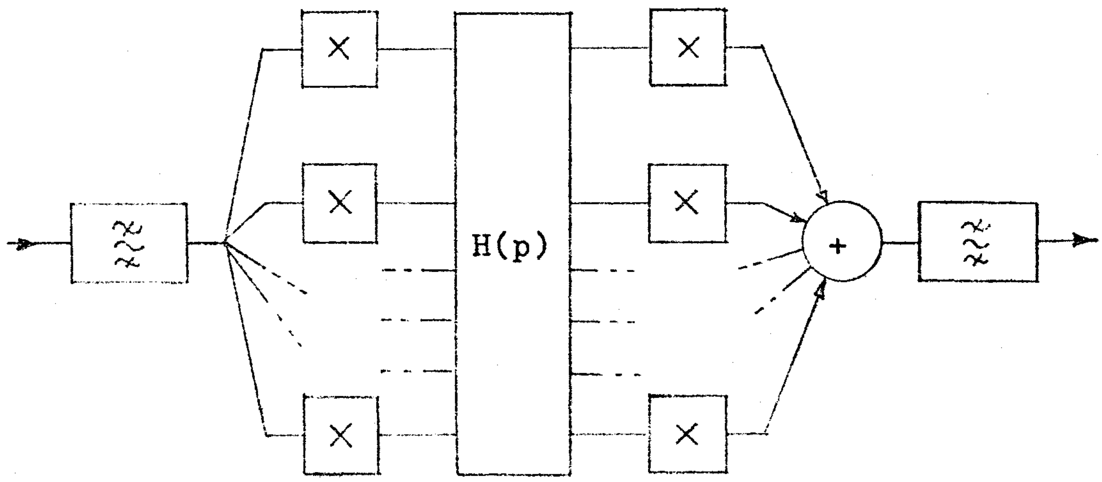


Figure 5.1.3 N-path Filter with Sequence Asymmetric Polyphase Filter.

The two modulator sets have applied to them sets of modulating signals of period T_1 and T_2 respectively. All the input modulators have the same waveform applied except that the waveform applied to the n th path is delayed in time with respect to that in the (n-1)th path by an amount T_1/xN where x is an integer and N is the number of paths. In the case of the second modulator set the delay is T_2/xN. Note: x = 1 or x = 2 for unbalanced or balanced modulators respectively.

Now the modulating functions r(t) and q(t) can be replaced by Fourier Series:

$$r(t) = \sum_{L=-\infty}^{L=+\infty} R_L \, e^{j\omega_1 Lt} \qquad \ldots\ldots (1)$$

where $$R_L = \frac{1}{T_1} \int_{-T_{1/2}}^{+T_{1/2}} r(t) \, e^{-j\omega_1 Lt} dt \qquad \ldots\ldots (2)$$

and $\omega_1 = 2\pi f_1 = 2\pi/T_1$

also $$q(t) = \sum_{K=-\infty}^{K=+\infty} Q_K \, e^{j\omega_2 Kt} \qquad \ldots\ldots (3)$$

where $$Q_K = \frac{1}{T_2} \int_{-T_{2/2}}^{+T_{2/2}} q(t) e^{-j\omega_2 Kt} dt \qquad \ldots\ldots (4)$$

and $\omega_2 = 2\pi f_2 = 2\pi/T_2$

Consider the nth path of the system as shown in Fig. 5.1.2. The following relations apply:

$$V_2(t) = V_1(t).r(t) \qquad \ldots\ldots (5)$$

$$V_3(t) = V_2(t).h(t) \qquad \ldots\ldots (6)$$

$$V_4(t) = V_3(t).q(t) \qquad \ldots\ldots (7)$$

$$\text{Vout} = \sum_{n=1}^{N} V_4(t) \qquad \ldots\ldots (8)$$

From (1) and (5)

$$V_2(t) = V_1(t). \sum_{L=-\infty}^{L=+\infty} R_L.e^{j\omega_1 Lt} \qquad \ldots\ldots (9)$$

Taking the Laplace transform

$$V_2(p) = \sum_{L=-\infty}^{L=+\infty} R_L.V_1(p-Lp_1) \qquad \ldots\ldots (10)$$

where $p_1 = j\omega_1$

At the output of the filter the signal becomes

$$V_3(p) = \sum_{L=-\infty}^{L=+\infty} R_L.H(p-Lp_1).V_1(p-Lp_1) \qquad \ldots\ldots (11)$$

And after the second modulator

$$V_4(p) = \sum_{\substack{L=-\infty \\ K=-\infty}}^{\substack{K=+\infty \\ L=+\infty}} R_L.Q_K.H(p-Lp_1).V_1(p-Lp_1-Kp_2) \qquad \ldots\ldots (12)$$

Summing the outputs of the N paths

$$V_0(p) = \sum_{n=1}^{n=N} V_4(p)$$

$$= \sum_{\substack{L=-\infty \\ K=-\infty \\ n=1}}^{\substack{n=N \\ K=+\infty \\ L=+\infty}} R_{Ln}.Q_{Kn}.H(p-Lp_1).V_1(p-Lp_1-Kp_2) \qquad \ldots\ldots (13)$$

Now as previously defined the modulating function in the nth path is delayed behind that in the (n-1)th path by a time T/xN being in all other respects identical.

Therefore $R_{Ln} = R_{Ln-1}.e^{-j(2\pi L/xN)}$

$$= R_{L1}.e^{-j2\pi L(n-1)/xN} \qquad \ldots\ldots (14)$$

Similarly

$$Q_{Kn} = Q_{K1}.e^{-j2\pi K(n-1)/xN} \qquad \ldots\ldots (15)$$

Now L, K, n, x and N are integers and considering the expression

$$\sum_{n=1}^{n=N} R_{Ln}Q_{Kn} = R_{L1}Q_{K1}.\sum_{n=1}^{n=N} e^{-j2\pi\left(\frac{K+L}{xN}\right)(n-1)} \qquad \ldots\ldots (16)$$

Let k+L = mxN and using the identity

$$1+a+a^2+\ldots\ldots+a^{N-1} = \frac{1-a^N}{1-a}$$

then (16) simplifies to

$$\sum_{n=1}^{n=N} R_{Ln}Q_{Kn} = R_{L1}.Q_{K1} \frac{\mathrm{Sin}(\pi mN)}{\mathrm{Sin}(\pi m)} \qquad \ldots\ldots (17)$$

$= N\, R_{L1}.Q_{K1}$ if m is an integer

= 0 if m is not an integer.

(Since if m is not an integer itself it must always at the least be the ratio of two integers).

The summed N path output (13) becomes therefore

$$V_0(p) = N \sum_{\substack{L=-\infty \\ K=-\infty}}^{\substack{K=+\infty \\ L=+\infty}} R_{L1} Q_{K1} \; H(p-Lp_1).V_1(p-Lp_1-Kp_2) \qquad \ldots\ldots (18)$$

The output therefore consists of a series of superimposed spectra each of which are the original signal modulated filtered and modulated again. The main terms of interest are normally when L = ±1 and K = ±1. All other terms can be supressed by pre and post band-limiting or by a suitable choice of modulating waveforms.

5.1.2 Use of N path Filters with Sequence Asymmetric Polyphase Filters

There is nothing in the foregoing analysis that precludes the use of filters which have a transfer function which is asymmetric about zero frequency. This can be done by substituting for the N filters a single N-phase sequence asymmetric polyphase filter as shown in Fig. 5.1.3. The filter will insert a transfer function of H(p) into each phase or path where H(p) may be asymmetric about zero frequency.

Now the input signal from the first set of modulators to each phase of the network is given in (10) as

$V_2(p) = \sum_{L=-\infty}^{L=+\infty} R_{Ln}.V_1(p-Lp_1)$ where R_{Ln} is the Lth term of the input modulator in the nth phase or path. Now from (14) with x = 1 $R_{Ln} = R_{L1}e^{-j2\pi L(n-1)/N}$

Combining these two equations and considering just one term in the signal spectrum the signal component on the nth phase is $a_n = R_{L1}.e^{-j2\pi L(n-1)/N}.V_1(p-Lp_1)$. The total N components on the N phases therefore make up a symmetrical polyphase set of vectors or a symmetrical component. The vectors are displaced by an angle $^{2\pi L}/_N$ apart and are rotating at an angular velocity of $2\pi(f-Lf_1)$ radians/sec. The components for $L = \pm 1$ will be the normal positive and negative sequences which will be correctly filtered by the types of filter described in Chapters 3 and 4. For other values of L this is not necessarily so and this must be taken into account when choosing the modulating function and band-limiting. In most cases unwanted products which will not be properly filtered will not combine with the output modulators to produce an unwanted interfering product.

Consider as an example the system shown in Fig. 5.1.4 where a 4 phase filter is used. The input and output modulators shown use series switches each of which is closed in turn for a quarter of the modulator period T_1. Applying the formula of eq.(2)

$$R_{L1} = \frac{1}{\pi L} \text{ Sin } \frac{\pi L}{4} \qquad \text{..... (19)}$$

The input signal set to the filter from the first modulator set is therefore a series of symmetrical components, one vector of one component being

$$a_n = \frac{1}{\pi L} \text{ Sin } \frac{\pi L}{4} \, e^{-j\pi L(n-1)/2}.V_1(p-Lp_1) \qquad \text{..... (20)}$$

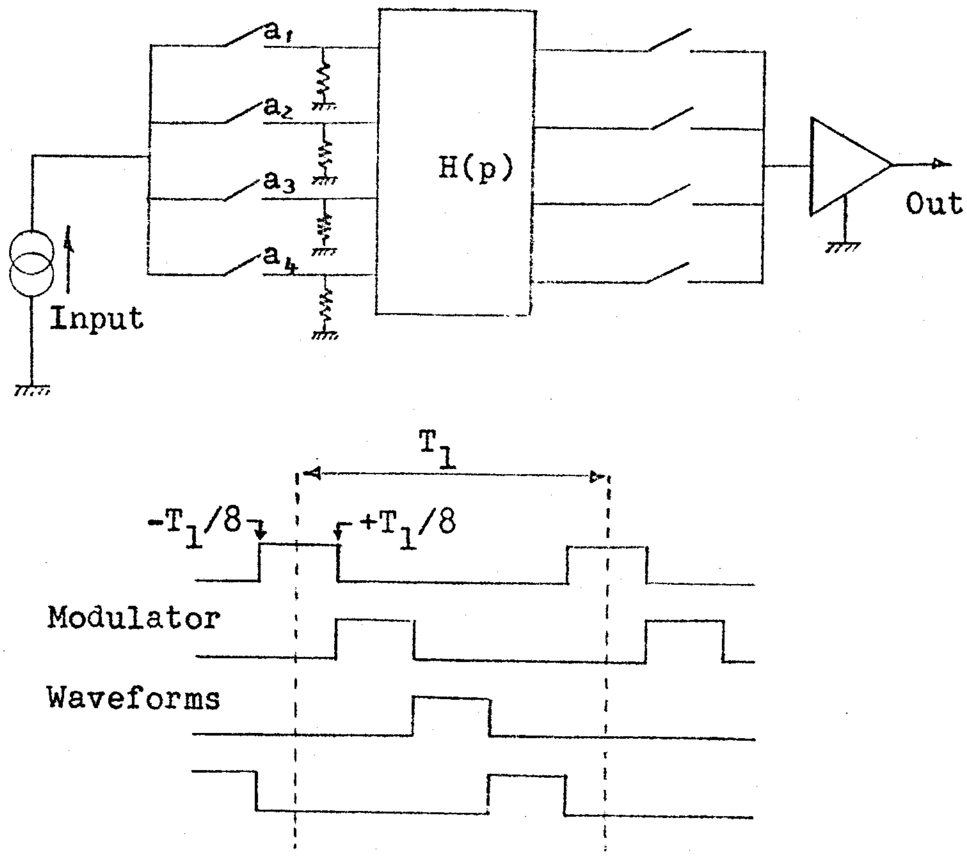


Figure 5.1.4 4-path Filter with Switch Modulators

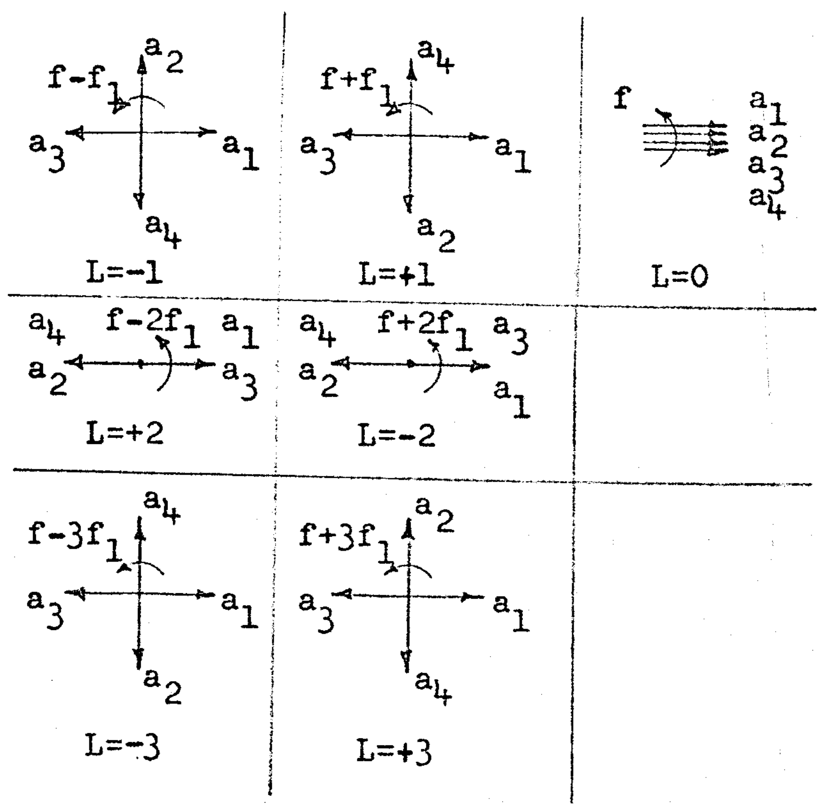


Figure 5.1.5 First few products generated by the input modulators in Fig 5.1.4

Figure 5.1.5 shows the first few products of modulation from L = 0 to L = ±3 for an input signal consisting of a sine wave of frequency f. This series of patterns will repeat for higher values of L. It can bee seen in this case that for odd values of L a positive or negative sequence is produced which would be correctly filtered by filters such as that of Fig. 4.3.6. The products produced by even values of L which are zero sequences will be filtered in a manner independant of positive or negative frequency. However since K + L = mN only certain values of K will give outputs corresponding to given values of L. The design of band-limiting filters must take into account what unwanted combinations of K and L are possible.

A simplified analysis results if the input and output modulators are supplied with purely sinusoidal waveforms as shown in Fig. 5.1.6 so that

$$r_n(t) = \text{Sin}\ (2\pi f_1 t + \frac{2\pi n}{N}) \qquad \text{..... (21)}$$

and $$q_n(t) = \text{Sin}\ (2\pi f_2 t + \frac{2\pi n}{N}) \qquad \text{..... (22)}$$

so that $$R_{L1} = \left.\begin{array}{ll} \frac{1}{2j} & \text{for } L = 1 \\ -\frac{1}{2j} & \text{for } L = -1 \\ 0 & \text{otherwise} \end{array}\right\} \qquad \text{..... (23)}$$

and $$Q_{K1} = \left.\begin{array}{ll} \frac{1}{2j} & \text{for } K = 1 \\ -\frac{1}{2j} & \text{for } K = -1 \\ 0 & \text{otherwise} \end{array}\right\} \qquad \text{..... (24)}$$

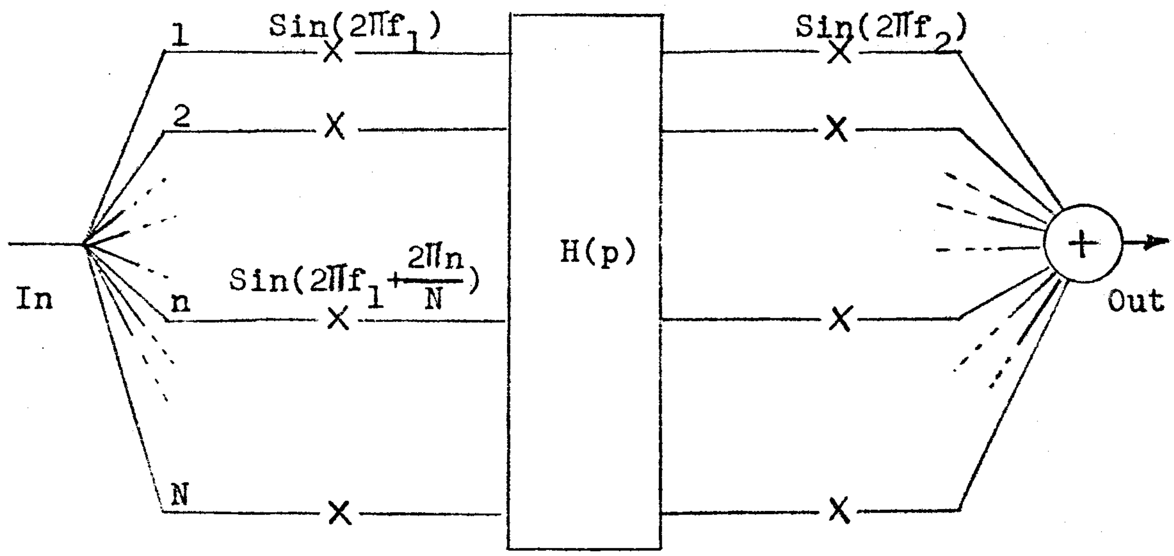
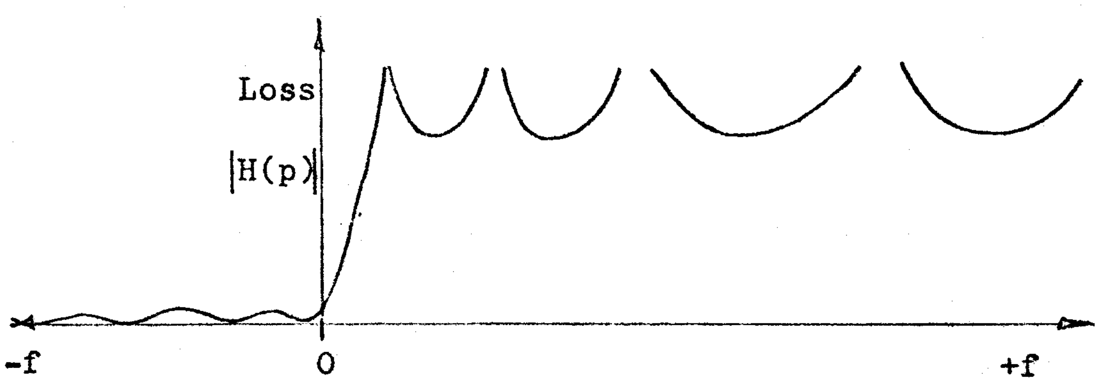
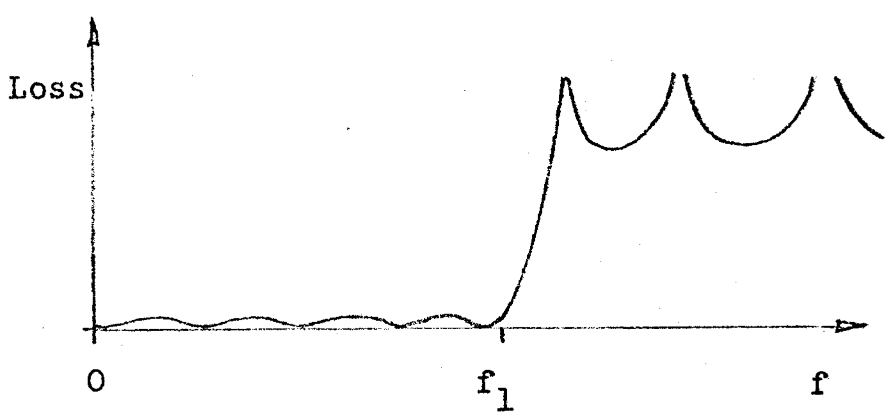


Figure 5.1.6 N-path Filter/modulator with sinusoidal modulating waveforms.



(a) Sequence asymmetric polyphase filter



(b) Response (a) shifted by modulation

Figure 5.1.7

The summed N path output (18) has then only two terms and becomes:

$$V_0(p) = \frac{N}{4}\left[H(p-p_1).V_1(p-p_1+p_2) + H(p+p_1).V_1(p+p_1-p_2)\right] \quad \ldots\ldots (25)$$

For an N path filter where $f_1 = f_2$ so $p_1 = p_2$ then (25) reduces to the transfer function

$$\frac{V_0(p)}{V_1(p)} = \frac{N}{4}\left[H(p-p_1) + H(p+p_1)\right] \quad \ldots\ldots (26)$$

This response is due to two separate polyphase signals passing through the filter, one at $f-f_1$ and one at $f+f_1$. By designing the filter transfer function H(p) to pass only negative frequencies and stop positive frequencies the second term in (26) can be made negligible and a tuneable lowpass filter results with a transfer function of:

$$\frac{V_0}{V_1} = \frac{N}{4}\ H(p-p_1) \quad \ldots\ldots (27)$$

A suitable type of filter transfer function is shown in Fig. 5.1.7 together with the effective response as an N path filter.

The quadrature modulation type low pass and highpass filters of Saraga (Ref. B6) are similar except that in that case phase splitting networks are used.

5.2 Direct Application to Single Sideband Modulation

An example was given in Chapter 1 of a single sideband modulator using a 4 phase sequence asymmetric filter. This will now be followed up in more detail with an analysis of the N-phase case. Filters of the type described in Chapter 4 are ideal for this application. Such filters can be made using only resistors and capacitors resulting in a very practical modulator or demodulator.

5.2.1 N Phase Symmetrical Components

The 4 phase case was discussed in Chapter 1 and the general principal can be extended to any number of phases. Proofs of the following formulae (28) and (29) appear in the book on Symmetrical Components by R. Neumann (Ref. A4).

Given an N phase system in which there are N arbitary signal voltages of the same frequency these N voltages can be represented as a set of N unsymmetrical vectors a'_n, n = 1.....N. This unsymmetrical vector set can be represented by a set of N symmetrical polyphase signals or symmetrical components. The superimposition or summing of these components will give the original unsymmetrical vectors.

Let the original unsymmetrical vectors be a'_n, n = 1.....N and the symmetrical components be a_r, r = 0 .....N-1 then they can be shown to be related by the following formulae

$$a_r = \frac{1}{N} \sum_{n=1}^{N} k^{(n-1)r} . a'_n \qquad ..... (28)$$

Conversely

$$a'_n = \sum_{r=0}^{N-1} k^{-(n-1)r}.a_r \qquad \text{..... (29)}$$

where $k = e^{j\phi}$ $\quad \phi = \frac{2\pi}{N}$

An individual symmetrical component has a voltage a_r on phase 1 of the network, $k^{-r}a_r$ on phase 2 and so on. On the nth phase the voltage will be $k^{-(n-1)r}.a_r$. This is depicted in Fig. 5.2.1. The total voltage on the nth phase is the sum of the contributions of the N symmetrical components as given in (29). Figure 1.2.3 in Chapter 1 shows a detailed example for N = 4.

5.2.2 Polyphase Modulation

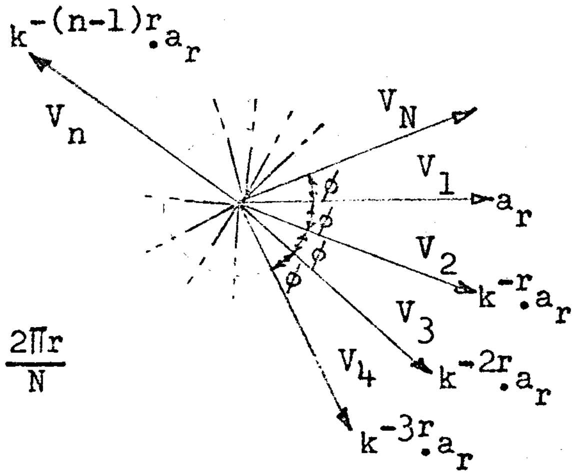
Consider a symmetrical component, the rth, applied to a set of N modulators one in each of the N phases as shown in Figure 5.2.2. Considering the nth phase an input signal V_n is multiplied in the modulator by a waveform $q_n(t)$ the output of which is summed with the outputs of the other modulators to produce the final output signal. As with the N path modulators in section 5.1 the modulating functions $q_n(t)$ can be replaced by Fourier Series:

$$q(t) = \sum_{K=-\infty}^{K=+\infty} Q_K e^{j\omega_c Kt} \qquad \text{..... (30)}$$

and $\quad Q_K = \frac{1}{T} \int_{-T/2}^{+T/2} q(t)e^{-j\omega_c Kt}.dt$

and $\quad \omega_c = 2\pi f_c$ and $T = 1/f_c$

f_c being the fundamental carrier frequency.



$$\phi = r\theta = \frac{2\pi r}{N}$$

$$k = e^{j\theta}$$

$$\theta = \frac{2\pi}{N}$$

Figure 5.2.1 rth. symmetrical component in an N phase set.

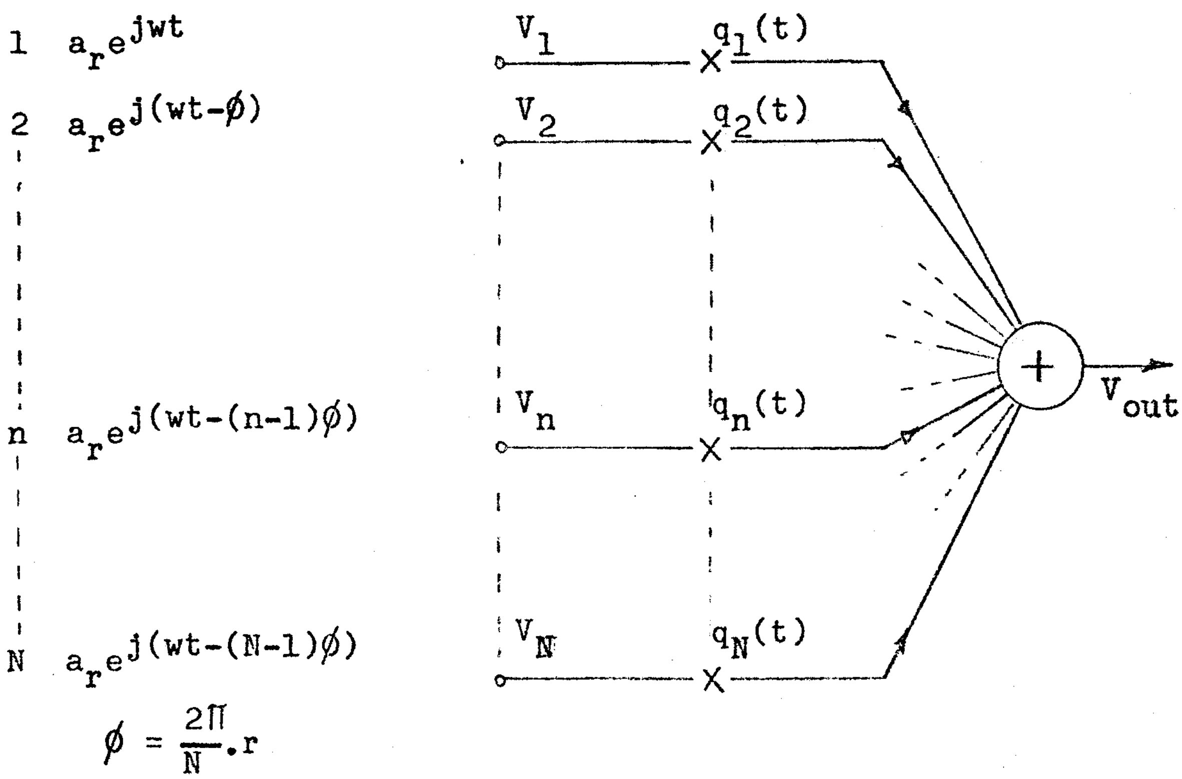


Figure 5.2.2 Polyphase modulator with a symmetrical component input signal.

Now if the modulating waveform in the nth path is delayed on that in the (n-1)th path by a time T/N being in all other respects identical then $q_n(t) = q_{n-1}(t-T/N)$

$$\text{and } Q_{Kn} = Q_{K1}.e^{-j2\pi K(n-1)/N} \qquad \ldots\ldots (31)$$

Also, if the input signal on the 1st phase is $V_1 = a_r e^{j\omega t}$ then if the input signal is the rth symmetrical component a signal of $V_n = a_r e^{j[\omega t-(n-1)2\pi r/N]}$ will exist at the input to the nth phase.

After modulation and summing the resultant products the output will be:

$$V_{OUT} = \sum_{n=1}^{N} V_n . q_n(t) \qquad \ldots\ldots (32)$$

$$= \sum_{n=1}^{N} \left[e^{[j\omega t-(n-1)2\pi r/N]} . \sum_{K=-\infty}^{K=+\infty} (Q_{K1} e^{-j2\pi K(n-1)/N} . e^{j\omega_c Kt}) \right]$$

$$= \sum_{K=-\infty}^{K=+\infty} \left[e^{j(\omega+K\omega_c)t} . Q_{K1} \sum_{n=1}^{N} e^{-j2\pi(n-1)(r+K)/N} \right] \qquad \ldots\ldots (33)$$

$$= N \sum_{K=-\infty}^{K=+\infty} Q_{K1} . e^{j(\omega+K\omega_c)t} \qquad \ldots\ldots (34)$$

if $K+r = mN$
m being an integer

$= 0$ otherwise

Thus, for a given value of r outputs occur for only certain values of K when $K = mN - r$ m being an integer.

In other words, r being the symmetrical component number, a given symmetrical component will give contributions in the output spectrum related to only certain values of K. Further, at the fundamental of the carrier frequency corresponding to $K = \pm 1$ only two components are possible. One, due to $K = +1$ being the upper sideband with $r = N-1$ $(m=1)$ and the other due to $K=-1$ being the lower or difference sideband with $r = 1$ $(m = 0)$. Other symmetrical components corresponding to the possible values of r between 0 and N-1 apart from the values 1 and N-1 contribute output sidebands only around d.c. and harmonics of the carrier frequency. Figure 5.2.3 gives the output spectrum for the case N=5 and shows which sidebands will be due to which symmetrical component.

5.2.3 Single Sideband Modulation

Single sideband modulation is the generation of one sideband about the carrier frequency while supressing the other. Now since in a polyphase modulator only one $(r=N-1)$ symmetrical components gives rise to the upper sideband and one $(r=1)$ gives rise to the lower sideband it is only necessary to supress one component with respect to the other to have a single sideband system. This is exactly what sequence asymmetric polyphase networks are designed to do. It is now clear, from the foregoing analysis that only their characteristics to the main positive and negative sequences are important. Other sequences give rise to products at harmonics of the carrier frequency only where they can be relatively easily eliminated. This

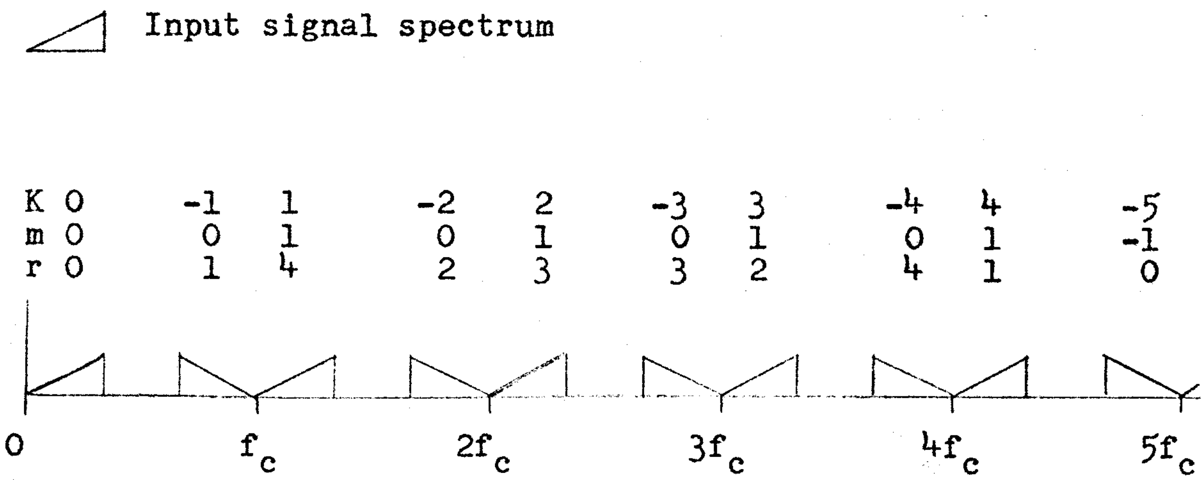


Figure 5.2.3 Output spectrum of polyphase modulator for N = 5. K+r = 5m

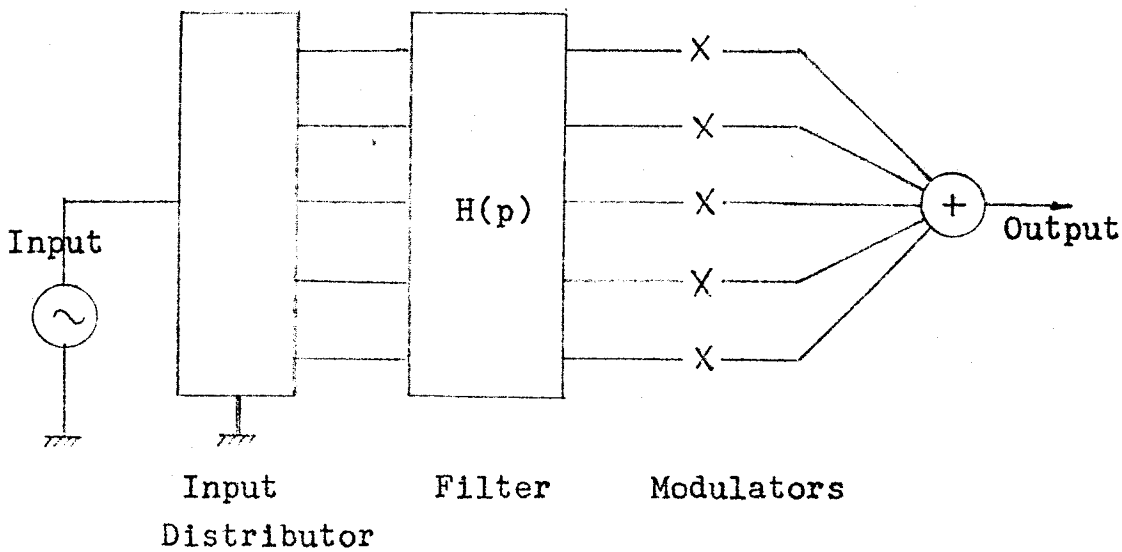


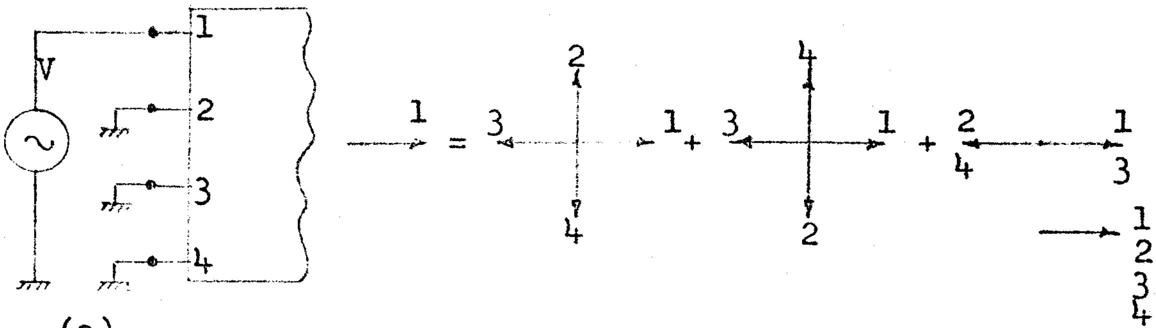
Figure 5.2.4 Polyphase Single Sideband Modulator with Input Signal Distributor.

is only strictly true for perfectly balanced modulators and filters although imperfections could be represented as modifications in the amount of each symmetrical component present.

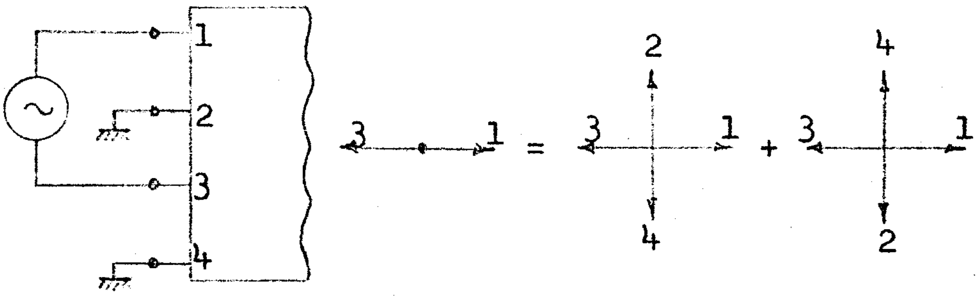
A complete N-phase single sideband modulator would look as in Fig. 5.2.4. A single input signal is distributed in some arbitary manner to the N inputs of a sequence asymmetric filter. The filter could be composed of a cascade of sections of the type shown in Fig. 4.2.7. The outputs of the filter are then applied to their respective modulators and the result summed.

The arbitarily distributed signal at the input impresses a voltage on each phase being some linear proportion of the input signal. This can be resolved into symmetrical components using eq.(28). Of these, one component is filtered by the positive frequency characteristic and produces only one sideband. A second component, filtered by the negative frequency characteristic produces only the other sideband.

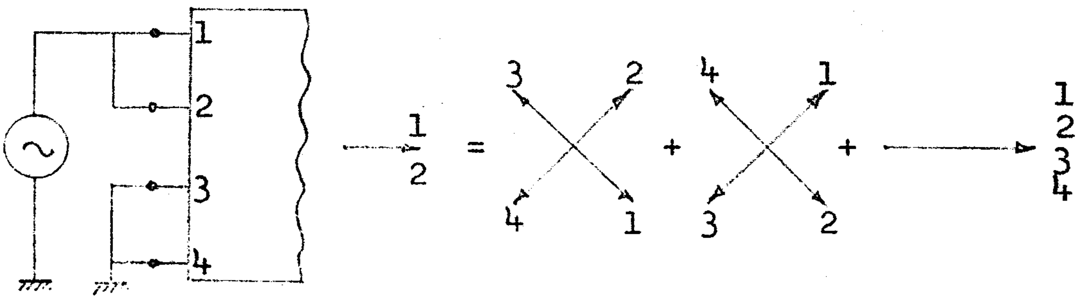
It is useful to connect the signal to the input to the filter so that a minimum of unwanted components result. Figure 1.4.3 in Chapter 1 showed such a case where, for N = 4, only the r = 1 and r = 3 components are produced. Usually when such a procedure is followed it will also be found that this also helps in minimizing the transmission loss. Figure 5.2.5 shows just three of the many other possible ways of driving a 4 phase filter.



(a)



(c)

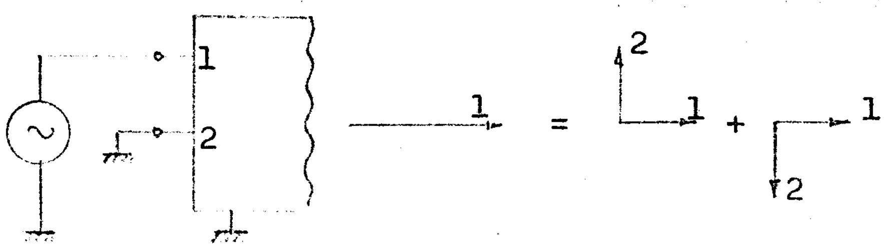


(b)

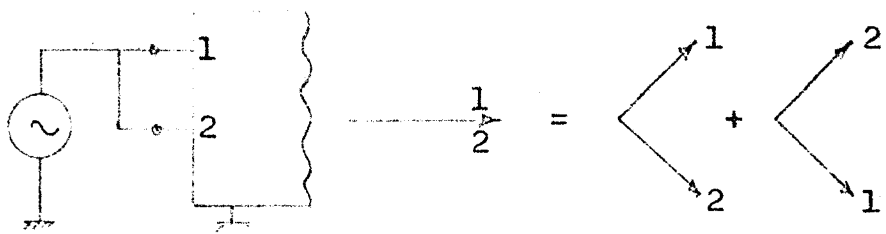
N.B. vectors not to scale.

(d) - see Fig. 1.4.3

Figure 5.2.5 Methods of driving a 4 phase filter.



(a)



(b)

Figure 5.2.6 Methods of driving a quadrature filter

Methods (a) and (b) are interesting because they show that it is possible to drive the filter from a single ended input.

A case of special interests occurs for the two-phase quadrature type of filters described in Chapter 3. These are a special case of the 4 phase filter and the method of symmetrical components can still be applied as shown in Fig. 5.2.6.

5.2.4 Practical Difficulties

The major practical difficulties with polyphase single sideband modulation are the following:

a) Loss of sideband suppression due to network inaccuracies.

b) Loss of sideband suppression due to gain and phase errors in the modulators.

c) Interference into and from other channels due to non-linearities in the modulators.

Quadrature modulation suffers from the same short comings being very similar in the way that sideband suppression is achieved by cancellation of modulation products. As has been discussed in Chapter 4 phase splitting type network inaccuracies are a much more severe problem than with polyphase sequence asymmetric filters.

Gain and Phase errors in the modulators can pose severe problems, particularly at high carrier frequencies.

The effect of such errors will now be discussed.

Suppose that in a polyphase modulator the signal has been filtered successfully so that at the input to the N modulators only the symmetrical component giving rise to the wanted sideband remains. If an error in gain or phase exists in one of the modulators it can be represented as a modification of the signal on that phase and the result broken into symmetrical components. The relative amplitude of the unwanted symmetrical component will then give the sideband discrimination.

For example, consider a 4 phase system where the input signals to the four modulators are:

$$a_1' = 1+\delta$$
$$a_2' = j$$
$$a_3' = -1$$
$$a_4' = -j$$

where δ is an amplitude error in phase 1.

Resolving into symmetrical components gives

$$a_1 = \tfrac{1}{4}\left[a_1'+ja_2'-a_3'-ja_4'\right] = \frac{\delta}{4} = \text{unwanted sideband}$$

$$a_3 = \tfrac{1}{4}\left[a_1'-ja_2'-a_3'+ja_4'\right] = 1 + \delta/4 = \text{wanted sideband}$$

$$\text{Sideband discrimination} = \frac{a_3}{a_1} = D$$

$$D = 20 \log_{10} \left(1 + \frac{4}{\delta}\right) \text{ dB}$$

eg. $\delta = .01 \quad D = 52.06\text{dB}$
$\delta = .001 \quad D = 72.04\text{dB}$

For analysis of a phase error ϕ the procedure is as before but with

$$a_1' = \text{Cos}\phi + j\text{Sin}\phi$$

then $$a_1 = \tfrac{1}{4}\left[\text{Cos}\phi - 1 + j\text{Sin}\phi\right]$$

$$a_3 = \tfrac{1}{4}\left[3 + \text{Cos}\phi + j\text{Sin}\phi\right]$$

and $$D = 10 \log_{10} \left| \frac{a_3}{a_1} \right|^2$$

$$= 10 \log_{10} \left[\frac{5+3\text{Cos}\phi}{1-\text{Cos}\phi}\right]$$

$\phi = 1^\circ \quad D = 47.2\text{dB}$

$\phi = 0.3 \quad D = 57.7\text{dB}$

$\phi = 0.1^\circ \quad D = 67.2\text{dB}$

In practise all the modulators will be in error in varying degrees and the foregoing analysis gives a guide as to what is acceptable in modulator performance. For example if 60dB minimum discrimination is required then a design target of $<0.1^\circ$ phase error and $<0.1\%$ amplitude error should be aimed for.

Phase error can be a serious problem at high carrier frequencies since while at 100kHz 0.1° represents a time error of 3nS at 10MHz it represents 0.03nS. While the former is a practical value with current technology the latter is not. It would therefore be prudent to employ a second stage of modulation if high carrier frequencies are desired. Following the publication by the Author of a brief paper on the subject of polyphase modulation (Ref. B12) some work has been done by others on the practical application using high carrier frequencies. This has met with some success since one worker (Ref. B14) reports a design using balanced integrated circuit modulators at 10.7MHz giving 50dB sideband suppression.

Modulators can also cause a certain amount of harmonic distortion and intermodulation. Intermodulation is usually the most serious since two tones A and B can intermodulate to form a term of the form $A \pm 2B \pm nf_c$ which can cause interference into other neighbouring channels. If the modulators are made with identical devices so that the harmonic distortion is identical then cancellation of some of the unwanted products of non linearity can be obtained. This is due to the following reasoning:

Suppose in an N phase system that a perfect symmetrical component is applied to the modulators as shown previously in Fig. 5.2.2. Now that component corresponding to say r = 1 will produce only the wanted sideband about the carrier fundamental frequency. Non linear transfer characteristics in the modulators can be effectively represented by a non linear element followed by a perfect modulator. If the non linear element has a transfer characteristic of the form

$$\frac{V_0}{V_{in}} = 1 + d_2V_{in} + d_3V_{in}^2 + d_4V_{in}^3 + \ldots..$$

then the output signal from the non linear elements will be

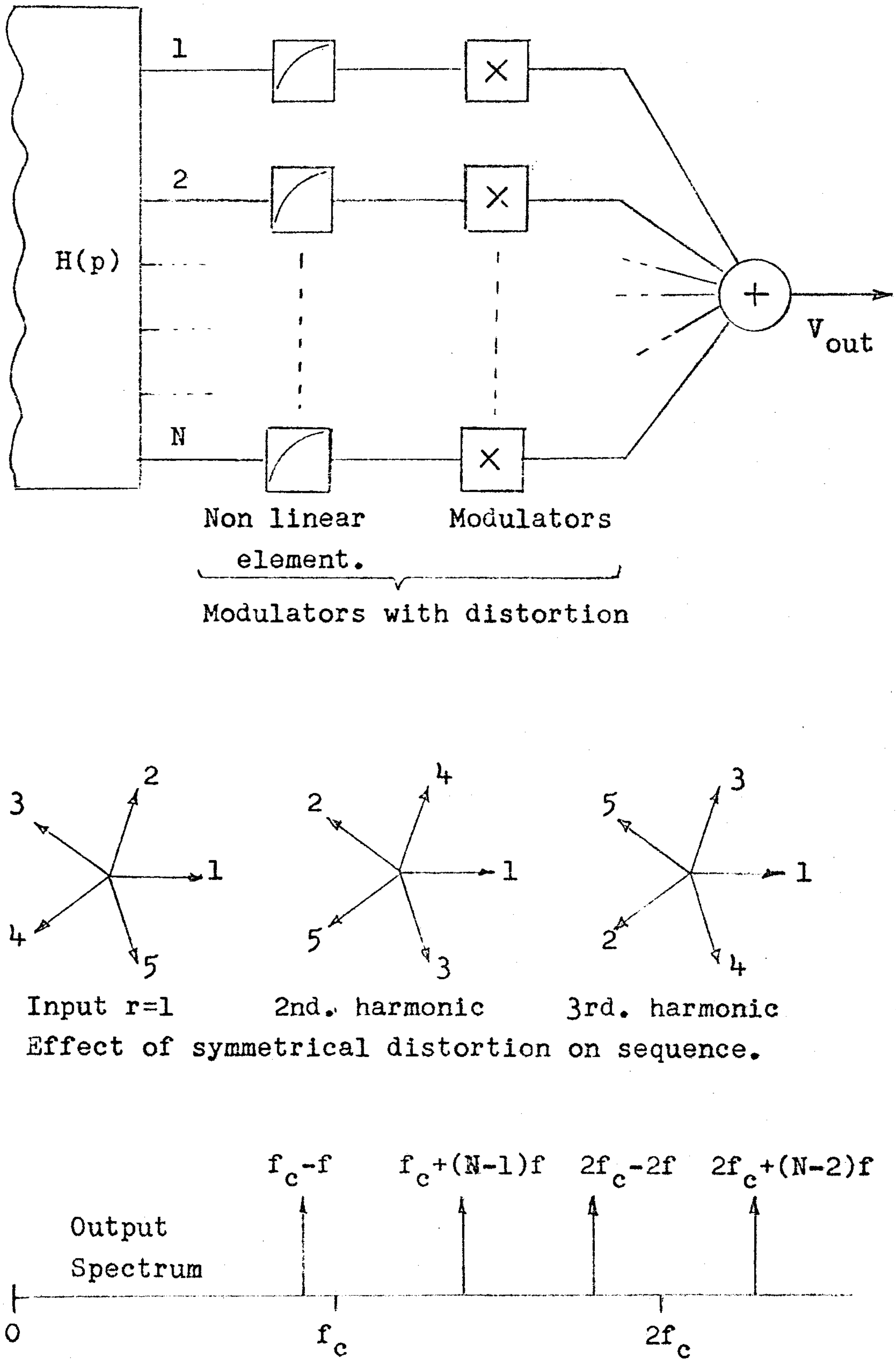


Figure 5.2.7 Modulator non linearities.

a symmetrical component r = 1, which is the original input, plus a series of components r = 2; 2nd harmonic, r = 3; 3rd harmonic etc. Thus the harmonics will not necessarily produce an output in the vicinity of the carrier frequency. The first one that will is the N-1th which will produce a $f_c+(N-1)f$ product ie. on the opposite side of the carrier to the wanted sideband. This is illustrated in Figure 5.2.7.

5.2.5 A Practical Single Sideband Modulator

A number of different SSB modulators and demodulators have been built using the principles given in this Thesis. This section describes a design which is a composite of several circuits actually built and illustrates the main practical points.

Figure 5.2.8 shows the complete schematic for a single sideband modulator and demodulator. The 4 phase RC polyphase networks are designed to give 62.5dB minimum sideband discrimination from 600Hz to 3400Hz. The modulators are MOS field effect transistors used as on-off switches controlled from a digital 4-phase supply. Such devices exhibit virtually infinite off resistance and a low on resistance from 30 to 1000 ohms depending on the type. In order to eliminate variations in on-resistance between each of the modulators in a set of four the circuit is designed so that the resistance plays no significant part in the transmission performance. This is achieved by working the modulators into a high impedance on the transmit side. Since no significant current flows through the modulators the variations in on resistance have little effect.

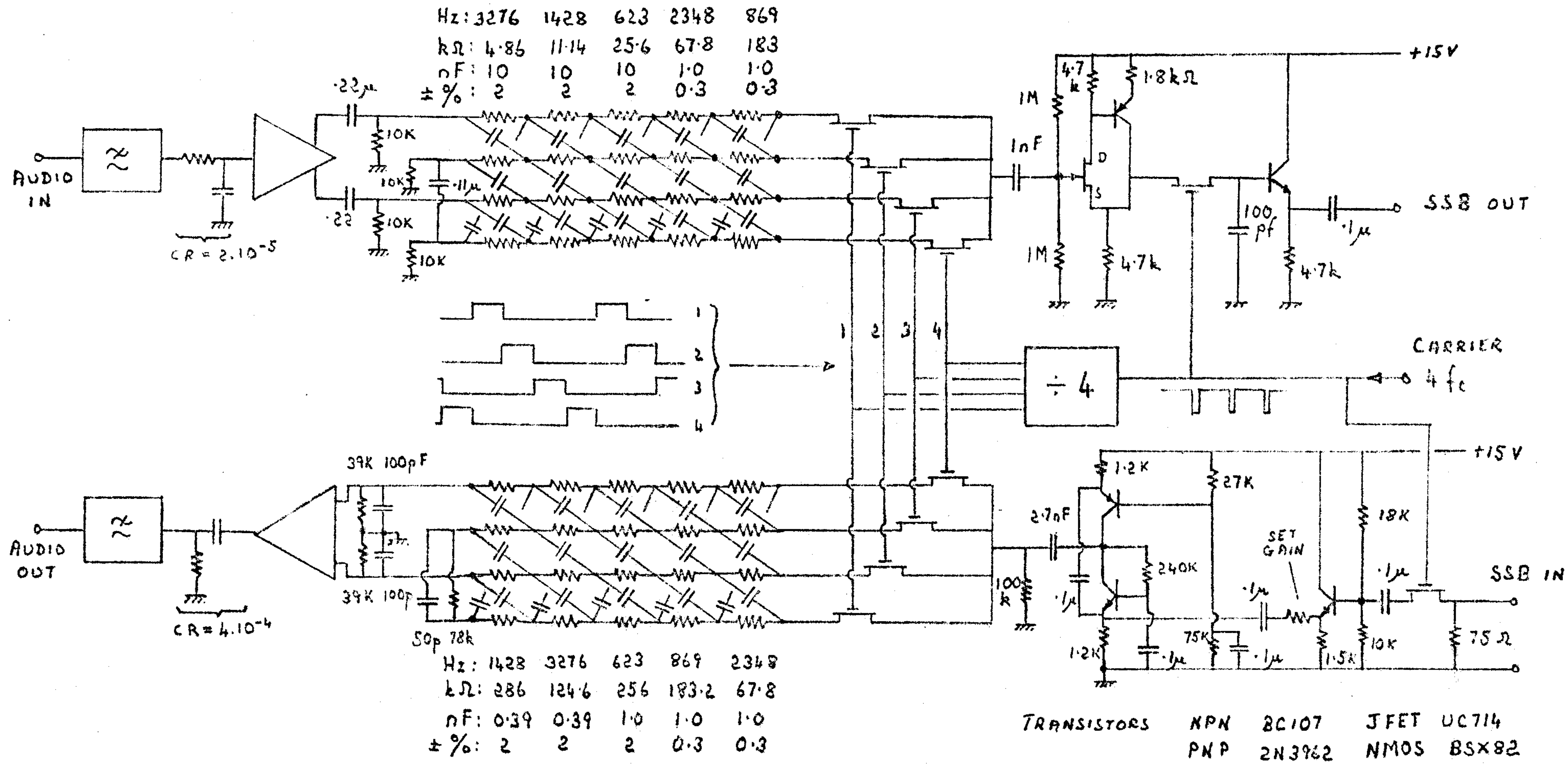


Figure 5.2.8 Complete Single Sideband Modulator and Demodulator.

The same effect is achieved on the receiver side by driving the modulators from a current source.

In addition, to reduce the effects of modulator timing variations a fifth switch is added in each direction. This switch is operated at 4 times the carrier frequency and arranged to chop out the signal during the period from when one modulator is about to switch off to when the next modulator has completed switching on. This also considerably reduces carrier leak which is largely due to variations in the transient spikes produced by the modulators.

The transmit polyphase filter is very similar to the design of Figure 4.3.6(e) discussed in chapter 4. Monte Carlo tolerance analysis indicates that 90% of networks should provide a minimum discrimination of 60dB. This is with component tolerances of ±2% for the first 3 sections and ±0.3% for the last 2 sections. Equalization of the passband response is provided by the RC low pass filter preceding the phase splitting network and the CR high pass coupling the amplifier into the filter input. An audio low pass filter precedes the transmit network to prevent signal frequencies above 3400Hz interferring into other channels.

The receive polyphase filter design is similar to the transmit design but in this case is reversed the highest accuracy components being at the input. Where as the transmit network is designed to work from a voltage source into an open circuit load the receive network is designed to work from a current source into a load resistor. This load resistor together with a shunt capacitor forms a low pass filter section at the output providing not only equalization at high audio frequencies but also preventing the following differential amplifier overloading. Without this filter adjacent channels would come through unchecked at frequencies above 4kHz causing intermodulation and noise. Low audio frequency equalization is provided by the CR high pass filter following the differential amplifier.

With careful attention to detail in the construction of the modulators this circuit is capable of giving the full sideband discrimination of 62.5dB. A similar design built with 70dB networks gave in excess of 68dB at a carrier frequency of 100kHz. Carrier frequency is a major factor limiting performance since it becomes progressively more difficult to achieve a good modulator performance at higher frequencies. A design giving 50dB sideband discrimination at 10.7MHz has been given in Ref. B14.

For good transmission performance linearity and intermodulation are critical. In particular the input amplifier on the demodulator may be called upon to handle a wide band of signal frequencies out of which the desired channel is to be selected. Intermodulation may cause two tones in other channels to form a product such as 2A±B which will fall in the wanted channel. The particular design in Fig. 5.2.8 gave intermodulation products which were more than 70dB down on the wanted signal.

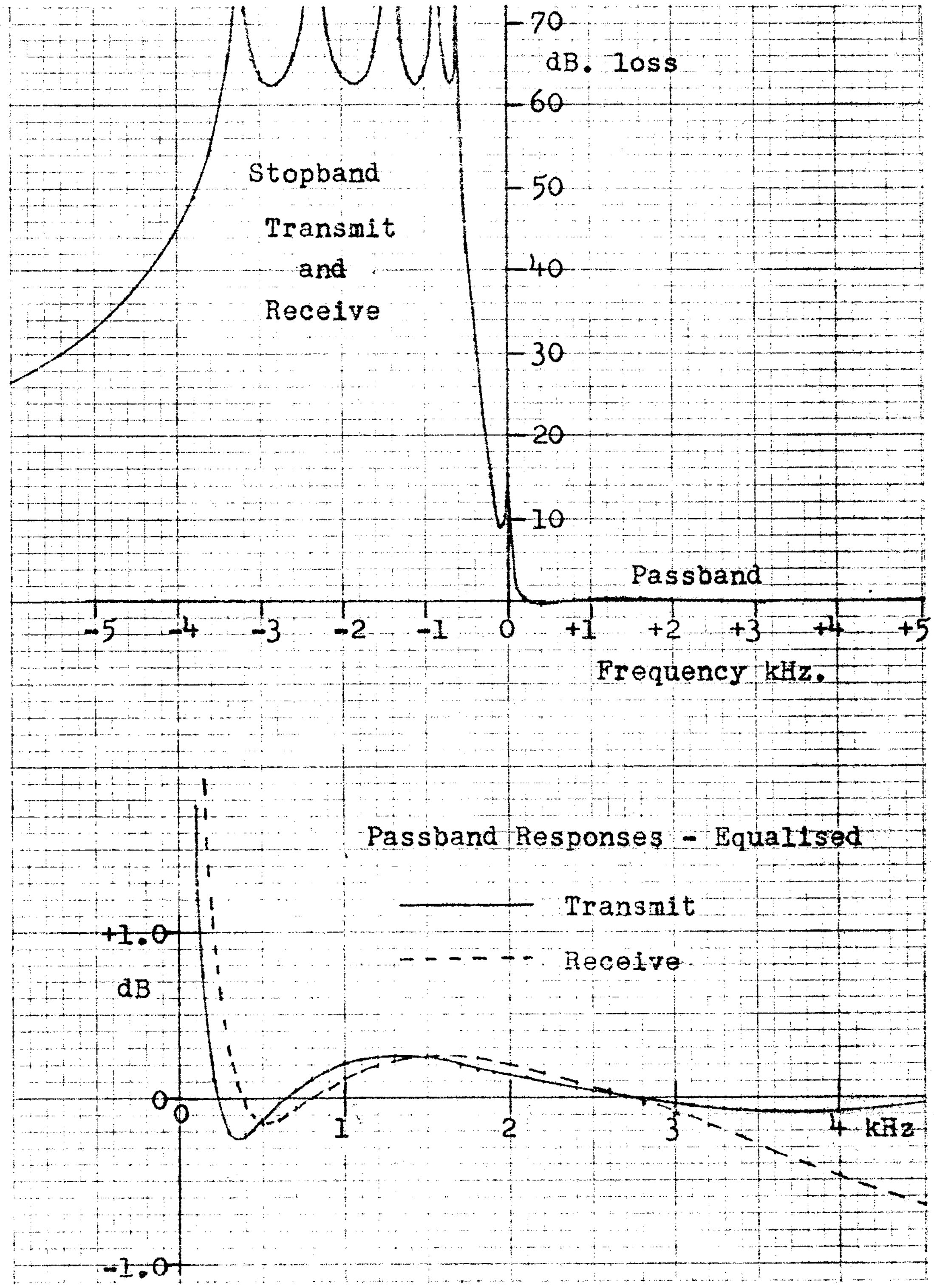


Figure 5.2.9 Frequency responses (computed) of the equalised polyphase filters used in Fig. 5.2.8

The frequency response in each direction was equalized to be <±0.2dB from 300 to 3400Hz. Complete frequency responses are given in Fig. 5.2.9 excluding the audio low pass filters.

Switching type modulators produce components at all the harmonics of the carrier frequency. It is therefore necessary to provide some band limiting at the transmit output and receive input.

5.3 Miscellaneous Applications

5.3.1 Phase Splitting and Combining

Polyphase networks can be used as phase splitters or combiners in any application which might previously have required a conventional two all pass phase splitting network. The advantages of lower component sensitvity and more convenient component values still apply.

An example application is in directional microphones (Ref. C12) where two microphones are mounted, one above the other, and oriented in quadrature. The electrical outputs are then combined through a phase combining network to give a desired directional response pattern.

5.3.2 Polyphase Oscillators

By connecting a polyphase filter in the feedback paths of a set of N amplifiers polyphase oscillators and sequence selective amplifiers can be constructed.

Figure 5.3.1 shows the circuit of a complete 4-phase oscillator constructed using a 4 phase R-C filter section in the feedback paths of a pair of differential amplifiers. The oscillation amplitude is controlled by a stabilizer circuit which places a very low impedance at the output of each phase of the feedback filter. In consequence the transmission characteristic of the filter is largely determined by the C parameter of the feedback network chain matrix.

A resistor R_1 in each phase was found to be necessary for stability and taking this into account the feedback network has the chain matrix

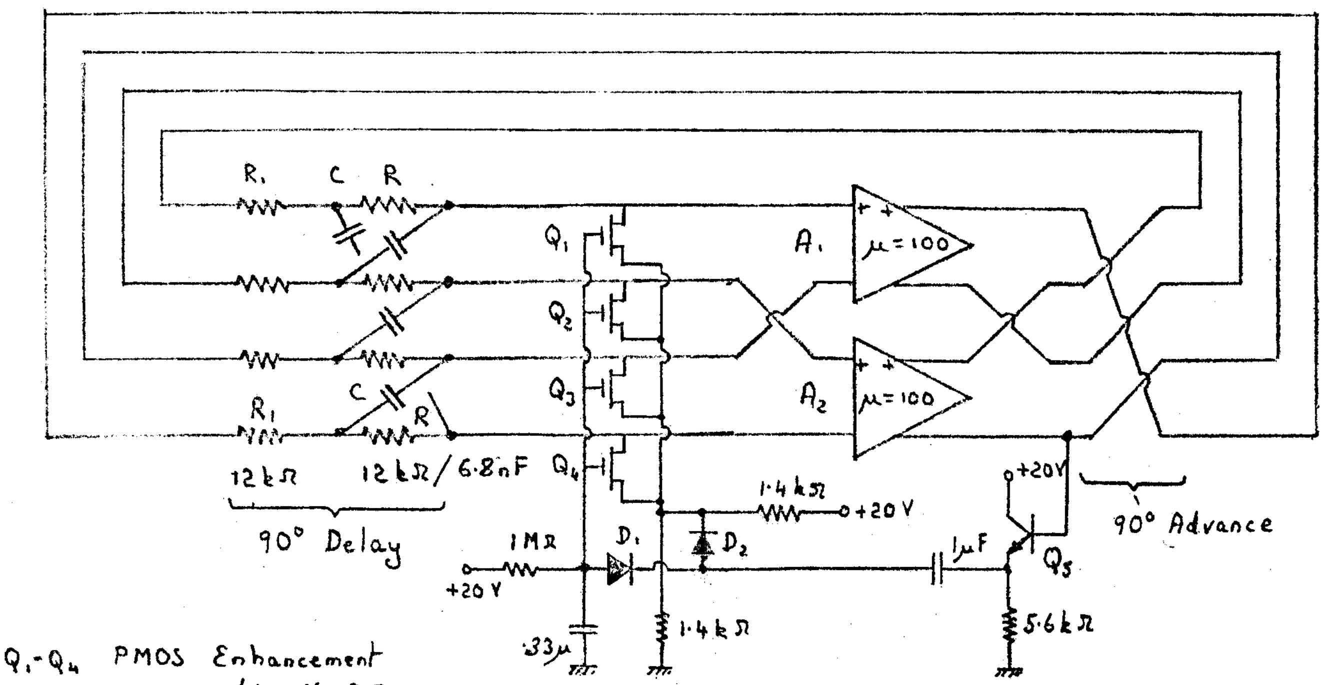
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{1+pC(R+R_1)}{1-\omega CR} & \frac{R+R_1+pCRR_1}{1-\omega CR} \\ \frac{2pC}{1-\omega CR} & \frac{1+pCR}{1-\omega CR} \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

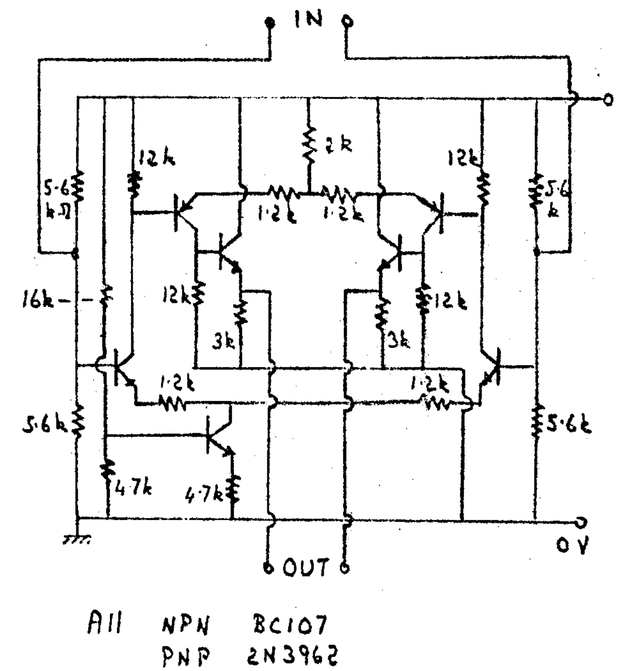
The gain regulating transistors Q_1-Q_4 present a very low impedance load resistance R_L so that the loop gain of the system would be approximately

$$G = \frac{2pC}{1-\omega CR} \cdot R_L \cdot \mu$$

where μ is the gain of the amplifiers (in this case $\mu = 100$). In order that the feedback network shall provide the maximum rejection of the unwanted sequence it is desirable that the circuit should oscillate at $\omega = -1/_{CR}$ and then the loop gain would be

$$G = \frac{-j\,\mu R_L}{R}$$





All NPN BC107
PNP 2N3962

Detail of one amplifier.

Q_1-Q_4 PMOS Enhancement
g_m 1.3mA/V V_T 3.5V

$$f = \frac{1}{2\pi RC}$$

Figure 5.3.1 A Four Phase Oscillator.

By skewing the amplifier outputs round by one phase the loop gain is made real. The gain regulation adjusts R_L automatically so that the loop gain is maintained just at unity.

This circuit exhibited good amplitude and frequency stability. The oscillation frequency was found to be 1871Hz and varied by ±1Hz for ±5 volts variation on the 20 volt power supply. The output amplitude was 4 volts peak to peak varying by ∓0.2dB for ±5 volts power supply change. Harmonic distortion was found to be very low, 2nd and 3rd harmonics being -63dB and -73.5dB relative to the fundamental respectively.

Amplitude and phase balance is dependant on the accuracy of the components used in the feedback network. In the model actually constructed the 4 phases were found to be matched in amplitude to .05dB and in 4 phase to 0.5^{o}.

5.3.3 Phase Balance Measurements

A polyphase filter constructed from accurate components can be used for making accurate phase measurements. By connecting an N phase signal to be measured to a polyphase filter of suitable design the positive and negative sequence components in the signal can be separated. The ratio of the amplitudes of the two sequences can give a very accurate measure of the phase balance of an N phase signal.

5.4 Complex Digital Filters

In recent years the design and application of complex digital filters has become important (Refs. B9-B11). Complex digital filters are filters which process complex data using complex coefficients. Since the complex data contains a real and imaginary component the filter can be realised in two paths one path processing the real data and one path the complex data. Cross connections between the paths provides the complex coefficients. This has been described by Crystal (Ref. B9) and Boite (Ref. B10).

By examining the structure of such filters and comparing them with the two phase quadrature filters of Chapter 3 it can be seen that they are very close duals of one another.

One problem posed by the published papers on complex filters is that of function synthesis. The only known published solution is that of a linear sideways frequency shift. This can be used but is likely to be inflexible. However, Chapter 2 of this Thesis gives a wide range of synthesis methods for asymmetric about zero transfer functions. These methods, when used in conjunction with the standard digital filter bilinear transform

$$\omega = \frac{1-Z^{-1}}{1+Z^{-1}} \qquad \text{with } Z = e^{j\omega T}$$

will give complex digital filter transfer functions suitable for most applications.

Complex digital filters designed using the foregoing principles can be used in digital single sideband modulators and N path filters in exactly the same way as has been described for analogue polyphase filters.