CHAPTER 4

Lossy Sequence Asymmetric Polyphase Filters

4.1 Introduction

It has long been known that it is possible to design passive R-C 90 degree phase splitting networks for use in single sideband modulation (refs. B1-B5). It seems reasonable, therefore, to expect that a modulation scheme using a passive R-C sequence asymmetric polyphase filter with equivalent performance is also possible. This indeed proves to be the case and the resulting polyphase network has some remarkable properties. Paradoxically no purely L-C polyphase filter equivalent to known L-C phase splitting networks has been found. Passive LC polyphase filters can be constructed however giving some unusual properties. This is discussed further in section 4.2.3.

Consider the circuit of Figure 4.1.1. This is a simple four phase R-C sequence asymmetric filter section. If the input of the network is driven by a perfect 4-phase signal as shown then the voltages and currents at all points in the network will also be in 4-phase. It is therefore possible to analyse the network considering only one path since what happens in that path also happens in the other paths shifted by 90^{o}, 180^{o} and 270^{o} respectively. Figure 4.1.2 shows all that needs to be considered to obtain the transfer function. The input and output conditions can be related by solving the circuit equations:

$$-jV_1 = \frac{I_x}{pC} + V_2$$

$$V_1 = (I_1 - jI_x)R + V_2$$

$$I_2 = I_1 - jI_x + I_x$$

Eliminating I_x and re-arranging gives the ABCD transmission matrix:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{1+pCR}{1+\omega CR} & \dfrac{R}{1+\omega CR} \\ \dfrac{2pC}{1+\omega CR} & \dfrac{1+pCR}{1+\omega CR} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

The frequency response, given in Figure 4.1.3 exhibits a peak of attenuation at $\omega = -{}^1/CR$ and a maximum gain at $\omega = +{}^1/CR$. The network is therefore sequence asymmetric.

The network of Figure 4.1.1 was in fact given in a paper published in 1950 (ref. C3) by Macdiarmid and Tucker. It was given in bridge form Figure 4.1.4 as a 'Sequence Discriminator suitable for use over narrow bands of frequency only'. A similar circuit for three phases was also given. In fact by cascading a number of such sections with suitably chosen resonant frequencies any bandwidth can be covered. Also by correctly choosing the order of the sections and their relative impedance levels the passband can be made extremely flat over a wideband of positive frequencies as shown in Figure 4.1.5.

Based on the foregoing principles many different types of asymmetric filter section are possible for any number of phases from three upwards. The so called two phase or quadrature arrangement is a degenerate form of a 4 phase network which would require transformers for its construction.

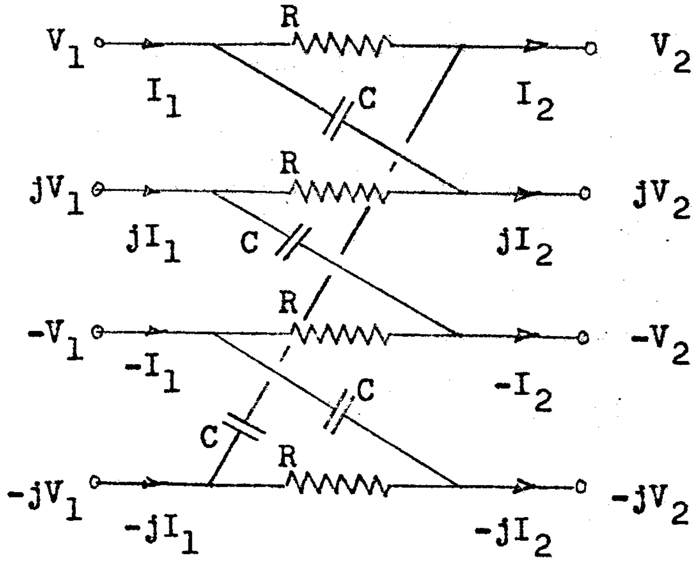


Figure 4.1.1 Four Phase R-C Filter Section.

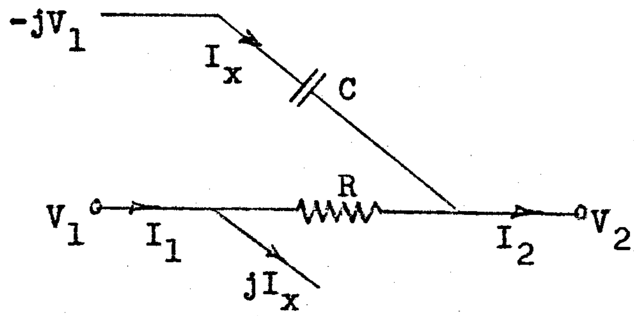


Figure 4.1.2 Analysis of a single phase.

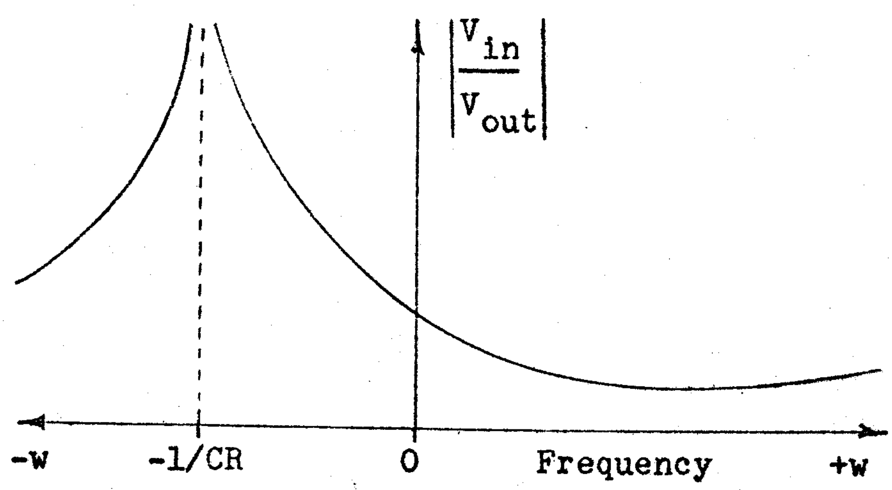


Figure 4.1.3 Loss characteristic of the network of Fig. 4.1.1

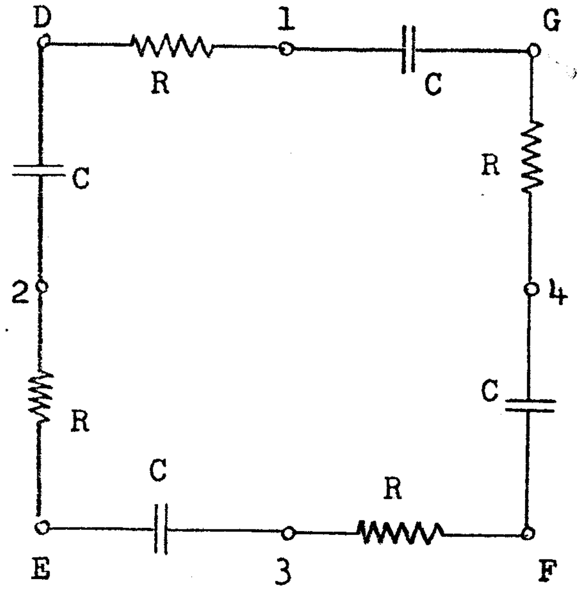


Figure 4.1.4 'Narrow Band' Four Phase

Discriminator given by Macdiarmid & Tucker.

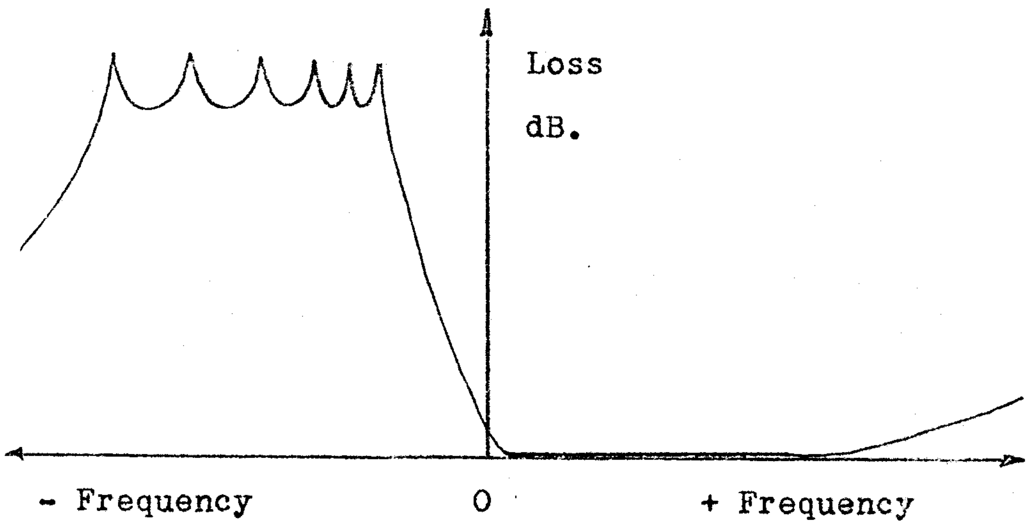


Figure 4.1.5 Practical frequency response

obtainable with filter using a cascade of N

sections of the type shown in Fig. 4.1.1

4.2 Types of Filter Section

4.2.1 The General N Phase Filter Section

The general N phase filter section would have N input nodes and N output nodes. Each input node would be connected to every output node via N branches requiring N^2 branches in all. In practise there is very little merit in going to this extreme and a much simpler network which covers most cases of practical interest is given in Figure 4.2.1. In this network the kth input node is connected to the kth output node by an admittance Y_1 and also to the (k+r)th output node by an admittance Y_2. As explained in section 4.1, because of symmetry, only one phase of the network has to be analysed to determine the transmission matrix. Figure 4.2.2 shows one phase from which can be obtained the equations relating input and output:

$$(C-jS)V_1 = (C-jS)I_x/Y_2 + V_2$$

$$V_1 = (I_1-I_x)/Y_1 + V_2$$

$$I_2 = I_1-I_x+(C-jS)I_x$$

where $C = \text{Cos}\left(\frac{2\Pi r}{N}\right)$

and $S = \text{Sin}\left(\frac{2\Pi r}{N}\right)$

Solving these equations, eliminating I_x and re-arranging gives the transmission matrix

$$\begin{bmatrix} V_1 \\ \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{Y_1+Y_2}{Y_1+(C-jS)Y_2} & \dfrac{1}{Y_1+(C-jS)Y_2} \\ \\ \dfrac{2(1-C)Y_1Y_2}{Y_1+(C-jS)Y_2} & \dfrac{Y_1+Y_2}{Y_1+(C-jS)Y_2} \end{bmatrix} \begin{bmatrix} V_2 \\ \\ I_2 \end{bmatrix}$$

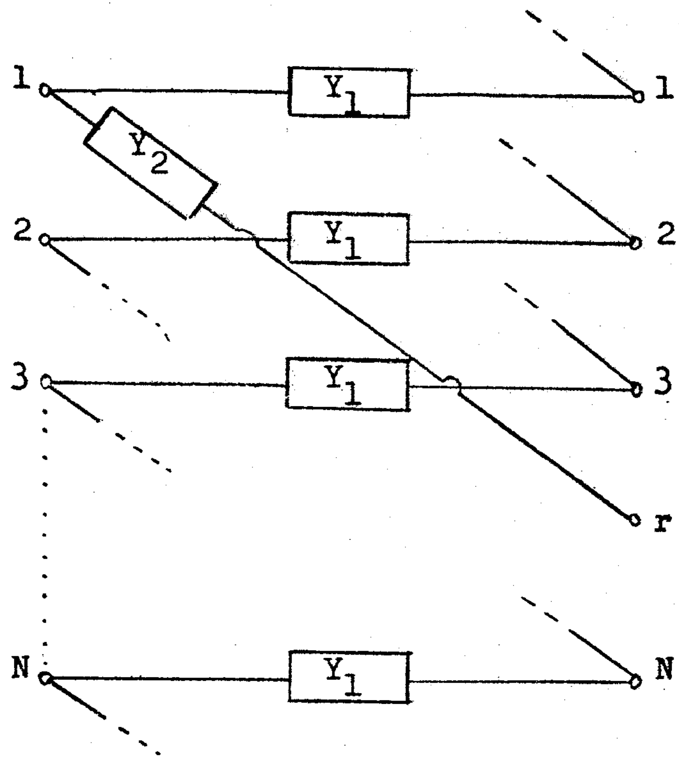


Figure 4.2.1 General N Phase Network.

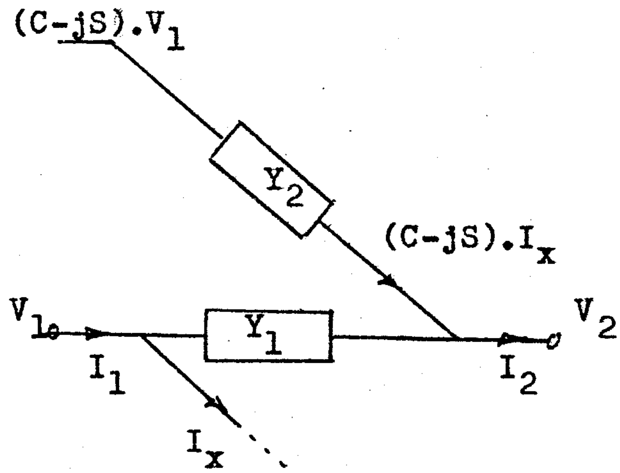


Figure 4.2.2 Analysis of a single phase of general N phase filter section.

$C = \mathrm{Cos}(\frac{2\pi r}{N}) \qquad S = \mathrm{Sin}(\frac{2\pi r}{N})$

For the 4 phase case already discussed in section 4.1 and shown in Figure 4.1.1 we have N=4 r=1

with $Y_1 = {}^1/R$ and $Y_2 = pC$

therefore $C = 0$ and $S = 1$

and the transmission matrix reduces to that already given in section 4.1.

The branches Y_1 and Y_2 can consist of any combination of capacitors coils and resistors. In this work the inclusion of coils has generally been regarded as disadvantageous because of their size and cost as compared to capacitors. Also they cannot be usefully integrated using present film technologies. Coils wound on ferrite cores do have one advantage because they usually have an adjustment facility. Accordingly sections with coils have been considered as possibly being useful in sections placed at the output end of a filter for single sideband modulation where high accuracy components would otherwise be required.

In general it appears most useful to choose Y_1 and Y_2 such that real transmission zeros are produced in the stop-band. For a given number of phases the relation between Y_1 and Y_2 is determined by the fact that at resonance

$$Y_1 + (C-jS)\, Y_2 = 0.$$

By choosing an appropriate structure for Y_1 then Y_2 is determined. For example in a three phase filter $C-jS = -\frac{1}{2}(1+j\sqrt{3})$ so that if Y_1 is a resistor then Y_2 will have to be a series or parallel combination of a resistor and a capacitor or coil to ensure a real transmission zero.

4.2.2 Three Phase Sections

Many different types of three phase section are possible and Figure 4.2.3 shows 4 types designed using resistors and capacitors only and for a real transmission zero at $\omega = {}^{1}/CR$. At the zero it is necessary to make the impedance of one branch equal to h times the impedance of the other branch (where $h = \frac{1}{2}(-1+j\sqrt{3})$).

Section B is virtually identical to section A since the same branches are transposed. The transmission matrix of section B is the same as that for A with all terms divided by h. Sections C and D are related to each other in the same way.

4.2.3 Four Phase Sections

Figure 4.2.4 shows five types of 4 phase section. Section A is a simple R-C type with a real zero at $\omega = -{}^{1}/CR$. Section B is the same as A but with the skew branches skewed in the reverse direction so that the real zero occurs at $+ {}^{1}/CR$.

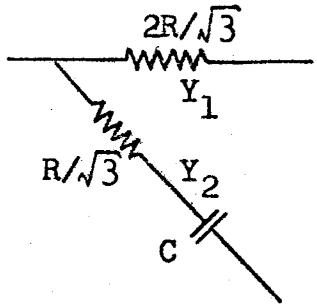
Section C using a coil and resistor is the dual of section A with an identical transmission matrix. Section D uses L, C and R provides two real transmission zeros.

Section E is unusual because it uses only coils and capacitors. Because of this the transmission zeros occur at complex frequencies. The image impedance is constant and resistive

$$Z_o = \sqrt{\frac{L_1}{2C_1}}$$

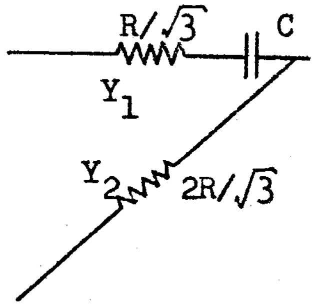
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Type A



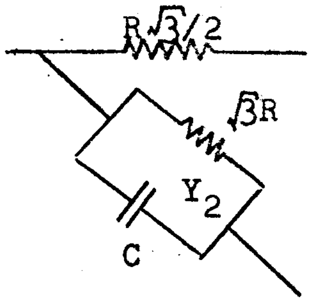
$$\begin{bmatrix} \dfrac{1 + j\sqrt{3}wCR}{1 + wCR} & \dfrac{2R(1+jwCR/\sqrt{3})}{3(1 + wCR)} \\ \dfrac{3pC}{1 + wCR} & \dfrac{1 + j\sqrt{3}wCR}{1 + wCR} \end{bmatrix}$$

Type B



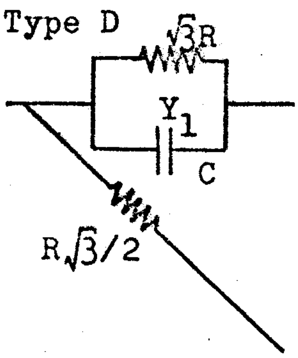
$$\begin{bmatrix} \dfrac{1 + j\sqrt{3}wCR}{h(1 + wCR)} & \dfrac{2R(1+jwCR/\sqrt{3})}{3h(1 + wCR)} \\ \dfrac{3pC}{h(1 + wCR)} & \dfrac{1 + j\sqrt{3}wCR}{h(1 + wCR)} \end{bmatrix}$$

Type C



$$\begin{bmatrix} \dfrac{pCR + \sqrt{3}}{-jh(1+wCR)} & \dfrac{R}{-jh(1+wCR)} \\ \dfrac{2(1+p\sqrt{3}CR)}{-jh(1+wCR)} & \dfrac{pCR + \sqrt{3}}{-jh(1+wCR)} \end{bmatrix}$$

Type D



$$\begin{bmatrix} \dfrac{pCR + \sqrt{3}}{j(1+wCR)} & \dfrac{R}{j(1+wCR)} \\ \dfrac{2(1+p\sqrt{3}CR)}{j(1+wCR)} & \dfrac{pCR + \sqrt{3}}{j(1+wCR)} \end{bmatrix}$$

$w=2\pi f$ $p=jw$ $j= -1$ $h= (-1 + j\sqrt{3})/2$

Figure 4.2.3 Three Phase Sections.

The image transfer function is given by $\log_e (F) = \log_e (A+\sqrt{BC})$ (since A = D) where F is given by

$$F = \frac{1-\omega^2 L_1 C_1 + j\omega\sqrt{2L_1 C_1}}{1+j\omega^2 L_1 C_1}$$

$$= j\frac{\left[\omega\sqrt{2L_1 C_1}+1-j\right]}{\left[\omega\sqrt{2L_1 C_1}+1+j\right]}$$

This forms a sequence asymmetric all pass filter. The poles and zero locations are restricted to lie on straight lines at 45° to the real frequency axis as shown in Figure 4.2.5. The transmission poles and zeros for the more usual sections A to D lie respectively on the imaginary and real axes of the p plane as shown in Figure 4.2.6. Since the LC all pass is a lossless network it should really have been included in Chapter 3 but as it was derived as a particular case of 4 phase lossy networks it has been presented here.

4.2.4. Sections with more than 4 phases

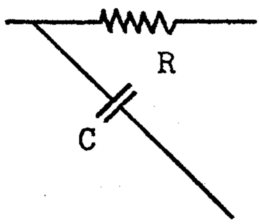
Sections, exhibiting a real transmission zero, with more than 4 phases can be made using one resistor in the 'Y_1' branch (Figure 4.2.2) and a series resistor and capacitor in the 'Y_2' branch. The Y_2 branch must be skewed over a sufficient number of phases so that at resonance a current balance is achieved at each output node.

At the transmission zero:

$$Y_1+(C-jS)Y_2 = 0$$

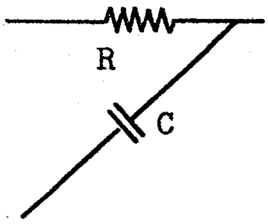
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Type A



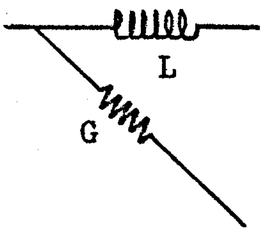
$$\begin{bmatrix} \frac{1+pCR}{1+wCR} & \frac{R}{1+wCR} \\ \frac{2pC}{1+wCR} & \frac{1+pCR}{1+wCR} \end{bmatrix} \qquad w_\infty = -1/CR$$

Type B



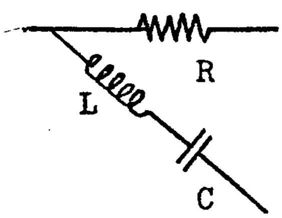
$$\begin{bmatrix} \frac{1+pCR}{1-wCR} & \frac{R}{1-wCR} \\ \frac{2pC}{1-wCR} & \frac{1+pCR}{1-wCR} \end{bmatrix} \qquad w_\infty = +1/CR$$

Type C



$$\begin{bmatrix} \frac{1+pLG}{1+wLG} & \frac{pL}{1+wLG} \\ \frac{2G}{1+wLG} & \frac{1+pLG}{1+wLG} \end{bmatrix} \qquad w_\infty = -1/LG$$

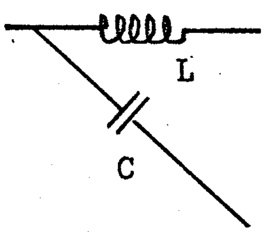
Type D



$$\begin{bmatrix} \frac{1+pCR+p^2LC}{1+wCR-w^2LC} & \frac{R(1+p^2LC)}{1+wCR-w^2LC} \\ \frac{2pC}{1+wCR-w^2LC} & \frac{1+pCR+p^2LC}{1+wCR-w^2LC} \end{bmatrix}$$

$$w_\infty = (CR \pm \sqrt{C^2R^2+4LC})/2LC$$

Type E



$$\begin{bmatrix} \frac{1-w^2LC}{1+jw^2LC} & \frac{jwL}{1+jw^2LC} \\ \frac{2jwC}{1+jw^2LC} & \frac{1-w^2LC}{1+jw^2LC} \end{bmatrix}$$

Figure 4.2.4 4 Phase Sections.

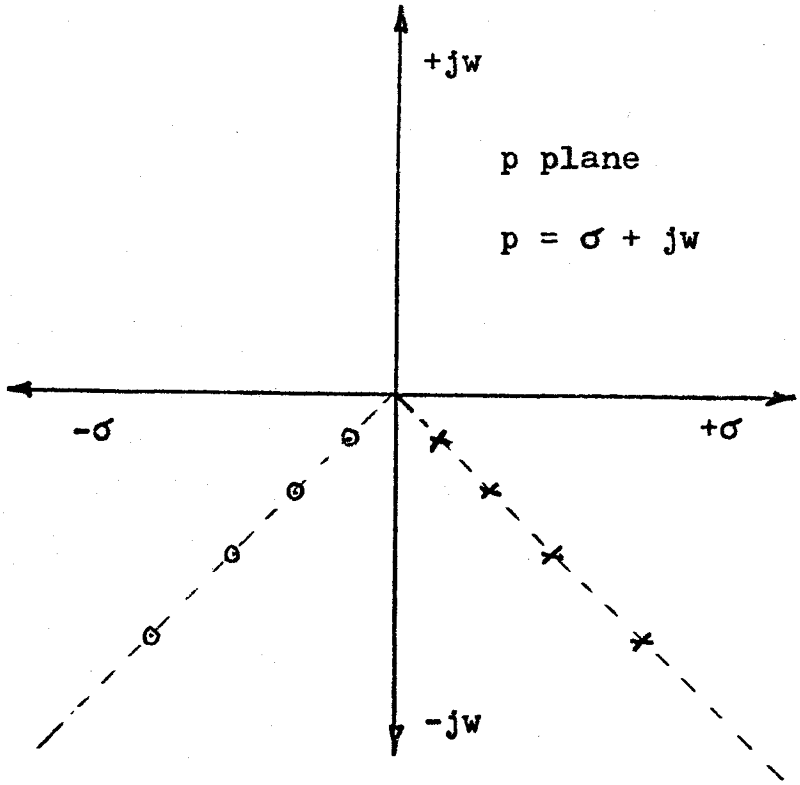


Figure 4.2.5 Pole and Zero locations for image terminated All-Pass section of Fig. 4.2.4E Poles and Zeros of F - see P.127 (top)

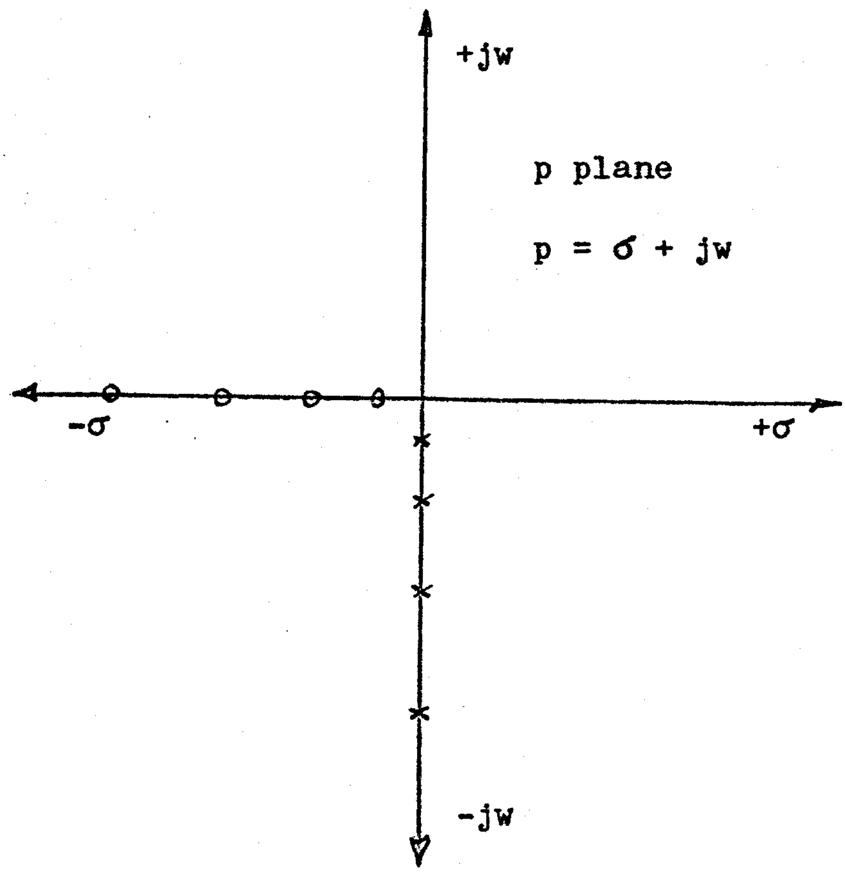


Figure 4.2.6 Pole and Zero locations for R-C Sequence Asymmetric Filters.

$$\text{or } Z_2+(C-jS)Z_1 = 0$$

$$\text{where } C = \text{Cos}\,\frac{2\pi r}{N} \qquad S = \text{Sin}\,\frac{2\pi r}{N}$$

If $Z_1 = R_1$ and $Z_2 = R_2 + \frac{1}{pC_2}$ then at resonance

$$R_2 + \frac{1}{j\omega C_2} + (C-jS)\,R_1 = 0$$

If $R_2 = -\,CR_1$ and C is negative ie. $\frac{\pi}{2} \leqslant \frac{2\pi r}{N} \leqslant \frac{3\pi}{2}$

$$N \leqslant 4r \leqslant 3N$$

then the resonance condition can be satisfied so that

$$\frac{1}{j\omega C_2} - jSR_1 = 0$$

$$\text{or } \omega\infty = \frac{-1}{S\,R_1C_2}$$

Figure 4.2.7 shows one phase of an N phase filter section designed to give a real transmission zero.

Figure 4.2.8 shows as an example how a five phase filter section would be made up. In this case r = 2 so that the skew branches are skewed across 2 phases.

4.2.5 Design of 4 phase sections with more than one transmission Zero

Figure 4.2.9 shows a more general version of the basic 4 phase filter section. Admittance branches connect every input node with every output node there being 16 branches in all.

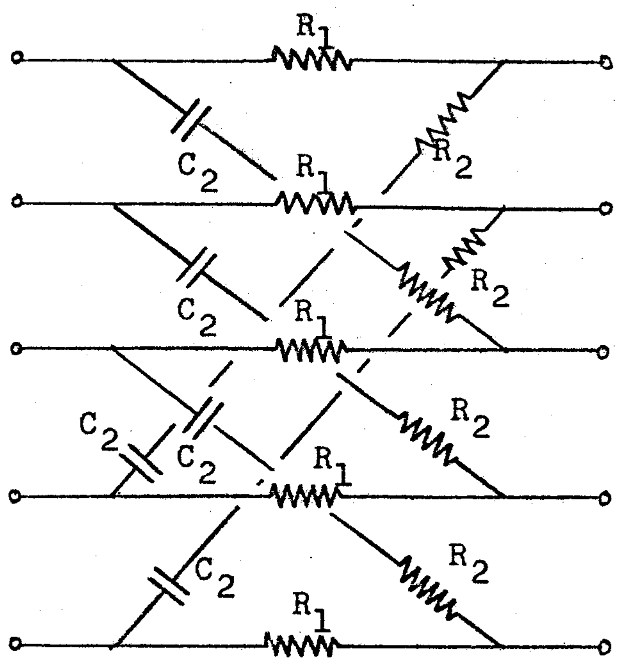
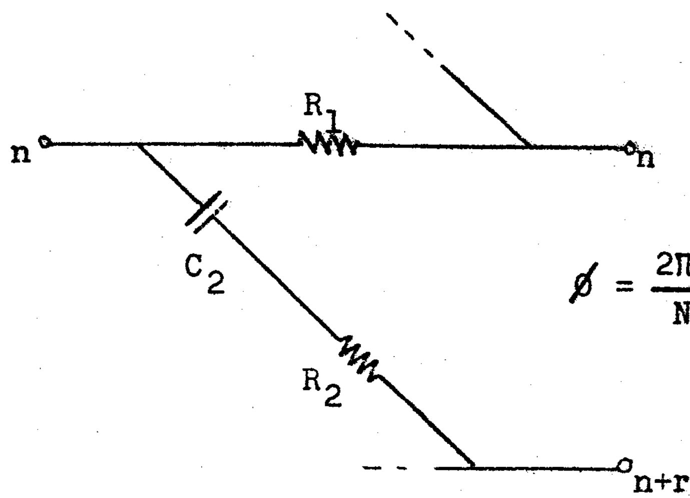


Figure 4.2.8 N-phase R-C Section.

N = 5 r = 2



$$\phi = \frac{2\pi r}{N}$$

$$R_1 = \frac{R}{\mathrm{Sin}\phi} \qquad R_2 = -R.\mathrm{Cot}\phi$$

$$w_\infty = -1/RC_2$$

Figure 4.2.7 One Phase of generalized R-C N-phase filter section.

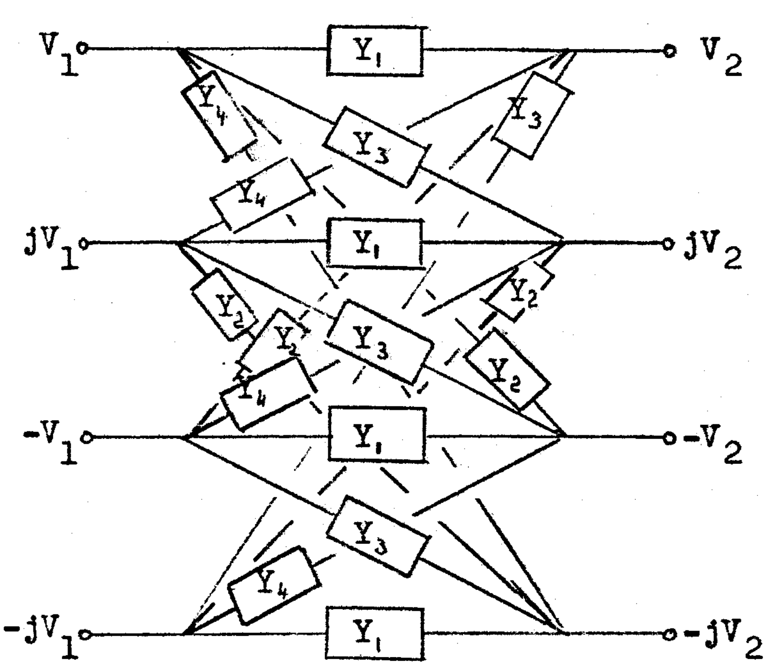
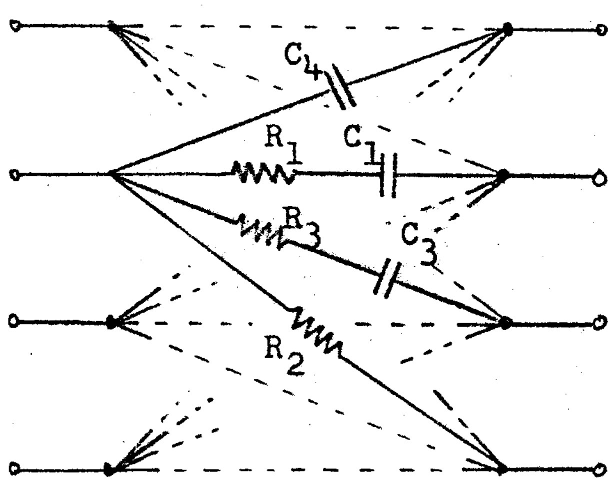


Figure 4.2.9 General 4-phase filter section.



$$R_1 = \frac{\gamma}{\alpha\gamma + \beta} \qquad C_1 = \frac{\alpha\gamma + \beta}{\gamma^2}$$

$$R_2 = \gamma/\beta \qquad C_2 = \frac{\gamma^2 + \beta}{\gamma\beta}$$

$$R_3 = \frac{\alpha}{\gamma + \beta/\gamma} \qquad C_3 = 1/\alpha$$

$$\alpha = \sqrt{\omega_1 + \omega_2 + \omega_3}$$

$$\beta = \sqrt{\omega_1 \omega_2 \omega_3}$$

$$\gamma = \sqrt{\omega_1\omega_2 + \omega_2\omega_3 + \omega_3\omega_1}$$

Figure 4.2.10 4-phase filter section with 3 transmission zeros.

Analysing the circuit, using the methods already described, the ABCD transmission matrix for this section is:

$$\begin{bmatrix} \dfrac{Y_1+Y_2+Y_3+Y_4}{Y_1-Y_2+j(Y_4-Y_3)} , & \dfrac{1}{Y_1-Y_2+j(Y_4-Y_3)} \\ \\ \dfrac{2(Y_1+Y_2)(Y_3+Y_4)+4Y_1Y_2+4Y_3Y_4}{Y_1-Y_2+j(Y_4-Y_3)} & \dfrac{Y_1+Y_2+Y_3+Y_4}{Y_1-Y_2+j(Y_4-Y_3)} \end{bmatrix}$$

If it is desired to design a section such as this for any number of real transmission zeros in the one section then the following general method can be used. Transmission is zero when the denominator term $Y_1-Y_2+j(Y_4-Y_3) = 0$.

Suppose, for example that a section with zeros at ω_1, ω_2 and ω_3 is required. Then $Y_1-Y_2+j(Y_4-Y_3)$ must be of the form $K(\omega-\omega_1)(\omega-\omega_2)(\omega-\omega_3)$ where K is arbitary.

Expanding:

$$Y_1-Y_2+j(Y_4-Y_3) =$$

$$= K\left[\omega^2+(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)\right]\omega$$

$$- K\left[\omega^2(\omega_1+\omega_2+\omega_3)+\omega_1\omega_2\omega_3\right]$$

Equating even and odd parts

$$Y_1-Y_2 = K\left[p^2(\omega_1+\omega_2+\omega_3) - \omega_1\omega_2\omega_3\right]$$

$$Y_4-Y_3 = Kp\left[p^2 - (\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)\right]$$

Let $\alpha = \sqrt{\omega_1+\omega_2+\omega_3}$

$\beta = \sqrt{\omega_1\omega_2\omega_3}$

$\gamma = \sqrt{\omega_1\omega_2+\omega_2\omega_3+\omega_3\omega_1}$

Then $Y_1-Y_2 = K(\alpha p-\beta)(\alpha p+\beta)$

$Y_4-Y_3 = Kp(p-\gamma)(p+\gamma)$

Now K can be chosen to give realiseable circuit elements Y_1 to Y_4

let $K = 1/(\alpha p+\beta)(p+\gamma)$

Then $Y_1-Y_2 = \dfrac{\alpha p-\beta}{p+\gamma}$

and $Y_4-Y_3 = \dfrac{p(p-\gamma)}{\alpha p+\beta}$

Which can be conveniently split into

$Y_1 = \dfrac{(\alpha+\frac{\beta}{\gamma})p}{p+\gamma}$ = resistor in series with capacitor

$Y_2 = \dfrac{\beta}{\gamma}$ = resistor

$Y_3 = \dfrac{(\gamma+\frac{\beta}{\alpha})p}{\alpha p+\beta}$ = resistor in series with capacitor

$Y_4 = {}^{p}/\alpha$ = capacitor

The final section is shown in Figure 4.2.10. Different solutions are possible by splitting Y_1-Y_2 and Y_4-Y_3 in different ways and also by the choice of K.

4.3 Complete Filters

4.3.1 Basic Considerations

In the preceding section, the transmission matrices for a number of different lossy filter sections are given and it can be seen that they can all be designed to give transmission zeros at any real frequency, negative or positive. The transmission poles are always imaginary with CR or LR filters and a complete filter would have the pole zero pattern already given in Figure 4.2.6.

In Chapter 2, subsection 2.3.3 a class of transfer functions was described giving equiripple passband, equiminima stopband performance. A special case was also described and this proves to be specially important since it is for a class of filters with transmission poles lying only on the imaginary axis which is exactly what is required for these RC and RL filter sections.

In practice direct design of high order filters is difficult and results in extreme component values. An alternative approximate method which takes advantage of the insensitivity of the filter and allows more realistic component values is described in section 4.3.3.

4.3.2 Exact Design from the Transfer Function

Before proceeding to design a practical filter it is necessary to know how it will be terminated. For single sideband modulation a convenient practical arrangement is to feed the network from a voltage source and terminate at the output in an open circuit. Thus we require only the 'A' parameter of the overall ABCD transmission matrix. This overall 'A' can be obtained by multiplying together the ABCD matrices of the individual sections.

No satisfactory direct synthesis method has yet been found. One procedure that has been considered for up to 4 sections is to equate the transfer function required with the transfer function of the actual network obtained by circuit analysis. Solving the equations gives the element values for the filter.

For example consider a 2 section 4 phase filter, each section being of the form of Fig. 4.2.4 (A) so that:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{1+pCR}{1+\omega CR} & \dfrac{R}{1+\omega CR} \\ \dfrac{2pC}{1+\omega CR} & \dfrac{1+pCR}{1+\omega CR} \end{bmatrix}$$

For two sections connected in cascade the overall ABCD matrix is

$$\begin{bmatrix} A^1 & B^1 \\ C^1 & D^1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$= \begin{bmatrix} A_1A_2+B_1C_2 \quad , & A_1B_2+B_1D_2 \\ C_1A_2+D_1C_2 \quad , & C_1B_2+D_1D_2 \end{bmatrix}$$

Hence $A^1 = A_1A_2+B_1C_2$

$$= \frac{(1+pC_1R_1)(1+pC_2R_2)+2pC_2R_1}{(1+\omega C_1R_1)(1+\omega C_2R_2)}$$

$$= \frac{p^2C_1R_1C_2R_2+p(C_1R_1+C_2R_2+2C_2R_1)+1}{(1+\omega C_1R_1)(1+\omega C_2R_2)}$$

Now in Appendix I tables are given of the poles and zeros for this type of transfer function in the form

$$\prod \frac{(p+\omega_Z)}{(\omega+\omega_p)}$$

It is therefore necessary to select the particular filter required for attenuation and bandwidth and obtain the transfer function.

$$H = \frac{p+\omega_{Z1}}{\omega+\omega_{p1}} \cdot \frac{p+\omega_{Z2}}{\omega+\omega_{p2}}$$

Note that because the tables are normalised about a midband of $\omega = 1$ then $\omega_{Z1} = 1/\omega_{Z2} = \alpha$

$$\omega_{p1} = 1/\omega_{p2} = \beta$$

Equating

$$H = \frac{p^2+p(\alpha+{}^1/\alpha)+1}{(1+\beta\omega)(1+\omega/\beta)}$$

$$H = \frac{p^2C_1C_2R_1R_2+p(C_1R_1+C_2R_2+2C_2R_1)+1}{(1+\omega C_1R_1)(1+\omega C_2R_2)}$$

therefore $C_1R_1 = \beta \qquad C_2R_2 = {}^1/\beta$

$$2C_2R_1 = \alpha + {}^1/\alpha - \beta - {}^1/\beta$$

therefore if $R_1 = 1$ then $C_1 = \beta$ and

$$C_2 = \tfrac{1}{2}\,(\alpha + \frac{1}{\alpha} - \beta - {}^1/\beta)$$

$$R_2 = \frac{2}{\beta(\alpha + \frac{1}{\alpha} - \beta - 1/\beta)}$$

Explicit formulae are therefore possible for two section filters. A similar procedure is possible for three and four section filters although the solution is required of a quadratic and a quartic equation respectively. Beyond four sections the algebraic manipulation required and the lack of convenient explicit solutions to equations above quartic in power makes this approach unprofitable. An alternative method which has been used sucessfully is to use a computer to automatically optimise the component values until the computed frequency response matches the theoretical transfer function within some pre-specified limits.

In practise computer optimisation is also unprofitable for the following reasons. The filter is made up of a number of sections each contributing one peak in the stopband. This determines the CR product for a given stopband performance. The only remaining action allowed to the computer optimization is to vary the relative impedance levels of filter sections. Due to the insensitivity of the filter to such variations it is necessary for the optimization to produce extreme ratios of section relative impedance level in order to get sufficient modifications of the frequency response to meet the theoretical passband performance. For example, in a six section 70dB filter, ratios between the resistors of the sections at the ends of the filter were found to be 3,000,000 to 1. Section 1 required 1 ohm resistors and 0.4162 F capacitors while section 6 required .376 micro-ohm resistors and 161,580 F capacitors. Because of the foregoing problems an approximate design procedure, allowing filters to be designed with a practical range of element values was developed.

4.3.3 An Approximate Design Procedure

In designing a passive polyphase filter the following parameters are under the control of the designer:

a) The number of sections and their resonant frequencies. This largely determines the stopband bandwidth and attenuation relative to the passband.

b) The relative impedance levels of the sections and section order. These have been found to be the main factors determining passband frequency response.

If N sections of any of the types so far described in this chapter are connected in cascade a sequence asymmetric filter will result. The transmission zeros will be determined, one per section, by the impedances used in each section. In a 4 phase section, of the type given in Fig. 4.2.4 (a) for example, the zero for that section will be determined solely by the R-C product of that section and will be independant of other sections. The transmission poles however are determined by the relative impedance levels of the sections or in other words by the interaction between sections. This is typified by the example in the preceding section 4.3.2 where the transmission poles of the two individual sections are modified by connecting the sections together and produce two new pole positions.

For these types of filter the transmission zeros all lie on the real frequency axis while the poles all lie on the imaginary axis. The synthesis of suitable transfer functions of this type is described in section 2.3.3 where the special case gives poles only on the imaginary axis. A typical filter taken from the tables given in Appendix 1 and having 6 sections with a stopband attenuation of 50dB minimum would have a pole-zero plot in the p-plane as shown in Figure 4.3.1. The frequency response of this filter, scaled to a mid-band of 805.6 Hz so that the stopband extends from -4000 Hz to -162.5 Hz, is given in Figure 4.3.2. This filter is typical of what might be required in many applications involving speech single sideband and will be used as a continuing example through the subsequent sections.

The interesting point to note about Figure 4.3.1. is that only the transmission zeros are asymmetric about zero on the real frequency axis. The transmission poles affect negative and positive frequencies by exactly the same amount.

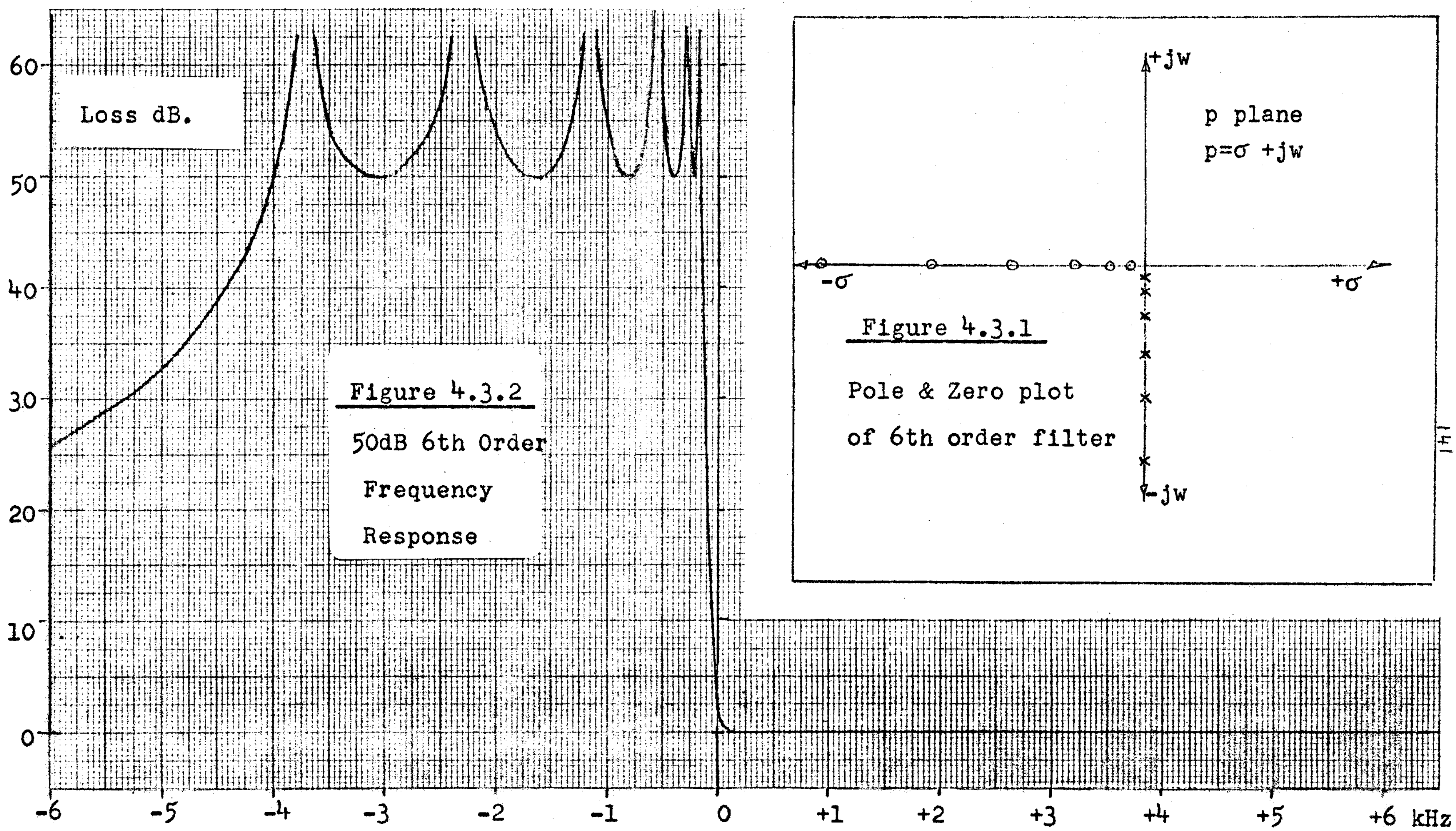


Figure 4.3.2
50dB 6th Order Frequency Response

Figure 4.3.1
Pole & Zero plot of 6th order filter

Figures 4.3.1 and 4.3.2

If one or more of the poles is moved along the imaginary frequency axis the discrimination at -f with respect to +f will remain unchanged. This can be shown by considering the transmission transfer function:

$$T(\omega) = \prod \frac{(\omega+\omega_Z)}{(\omega+j\omega_p)}$$

and $$T(-\omega) = \prod \frac{(-\omega+\omega_Z)}{(-\omega+j\omega_p)}$$

Discrimination at ω with respect to $-\omega$

$$= \frac{T(\omega)}{T(-\omega)} = \prod \frac{(\omega_Z+\omega)}{(\omega_Z-\omega)} \cdot \frac{(j\omega_p-\omega)}{(j\omega_p+\omega)}$$

$$|\text{Discrimination}| = \prod \frac{\omega_Z+\omega}{\omega_Z-\omega}$$

- independant of ω_p

If a filter is designed using values of ω_Z chosen to give some arbitary discrimination then that discrimination will be maintained regardless of the choice of the values of ω_p. In practice this means that a filter can be designed using values of ω_Z, taken from the tabulated functions in Appendix 1, and providing the passband is made flat by some means which is symmetrical about zero frequency then the stopband will have its correct equi minima shape and the discrimination will be maintained. The final response will differ from the theoretical response in the following respects:

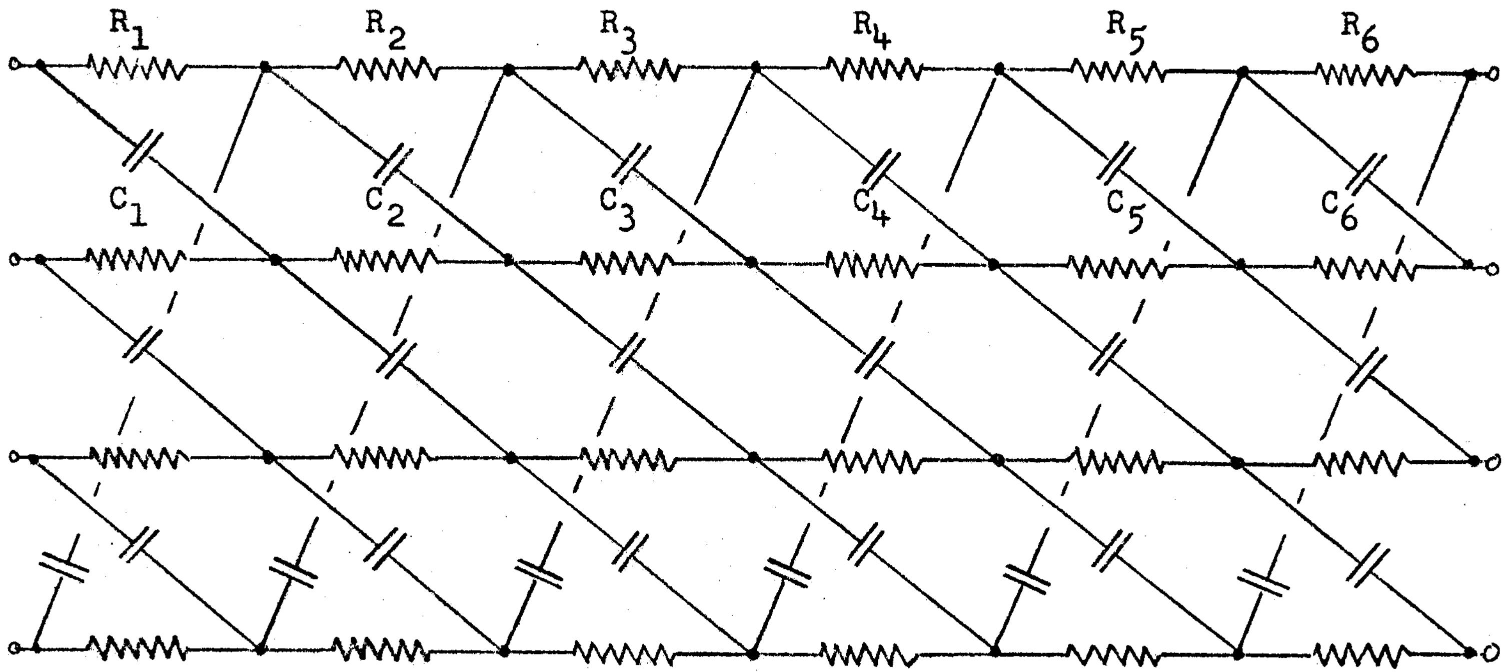
a) The passband ripple does not have to be exactly the same. This would generally be unnecessary anyway as the theoretical ripple is far smaller than is normally required. For example a 50dB stopband design gives a theoretical passband ripple of .00004 dB. The effect of choosing a ripple up to 0.5dB would be relatively unimportant with a 50 dB stopband.

b) The absolute loss will be shifted by some amount depending on the equalisation needed.

c) Phase shift and hence group delay may be different.

The passband can be made flat by modifying the relative impedance levels, choosing the optimum section order and using simple R-C equalisers that affect negative and positive frequencies equally. This will be made clear in the following examples which use the 6th order 50dB stopband transfer function of Figure 4.3.2 as a design objective.

Figure 4.3.3 shows a practical 6 section 4 phase RC filter structure capable of providing the frequency response given in Figure 4.3.2. Also given in Figure 4.3.3 are the component values required to meet the theoretical transfer function. These values were obtained by a computer optimization programme which strives to make the practical frequency response match the theoretical response within very close limits (less than .03dB in this case). To meet the ideal



R kΩ	1.0000	2.6929	10.239	43.097	132.86	401.25
C nF	42.7335	25.602	13.385	6.608	4.261	2.276
F kHz.	3.7244	2.3085	1.1610	0.5589	0.2811	0.1743

Figure 4.3.3 Six section 50 dB 4 phase sequence asymmetric filter. Component values optimised to match theoretical transfer function.

Ph.D. Thesis "The Synthesis and Application of Polyphase Filters with Sequence Asymmetric Properties"
Michael John Gingell 1975 University of London Faculty of Engineering.

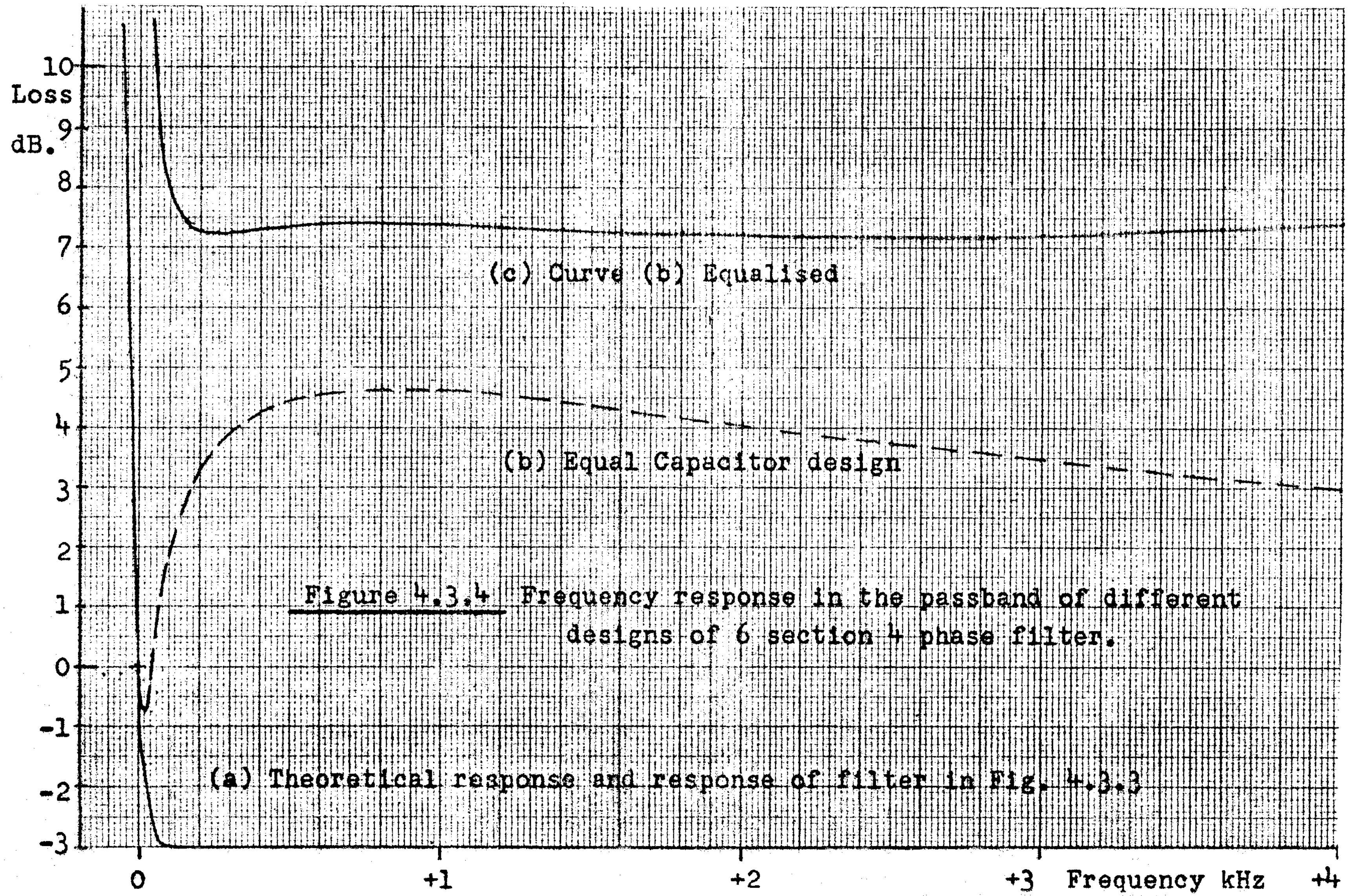


Figure 4.3.4 Frequency response in the passband of different designs of 6 section 4 phase filter.

response it is necessary to drive the network from a voltage source and to terminate with an open circuit. To do otherwise would cause an increase in basic loss. Each section produces one transmission zero determined by the R-C product of that section and many different designs are possible simply by varying the order of sections. The filter in the case of Fig. 4.3.3 has the sections in order of zero frequency, highest at the input falling to lowest at the output. This design has a 400 to 1 variation in resistor values and 20 to 1 variation in capacitor values. Suppose we make all the capacitors have the same value and adjust the resistors to give the correct resonant frequencies. The result of this is shown in Figure 4.3.4 curve (b) where the passband loss has increased by 7 to 8dB compared with the ideal response (curve (a)). The shape of the response is also modified having a loss of -0.7dB at 50Hz rising to a maximum loss of 4.6 dB at 800Hz and then falling steadily to 0dB at infinite frequency. This kind of shape can easily be equalised with two R-C time constant - a CR high pass filter to lift the loss at low frequencies and an RC low pass filter to do the same at high frequencies. The result of such an equalization is shown in Figure 4.3.4 curve (c) where it can be seen that the response has been equalised to give a peak to peak ripple of 0.2dB from 160Hz to 4kHz. The high pass filter can be combined into the input of the polyphase filter where it performs the very useful secondary function of blocking d.c. from any active input circuitry that might be used. Where these filters are combined with modulators the presence of d.c. can cause unwanted carrier leak and other problems which are circumvented by the high pass filter. A suitable drive circuit for single sideband modulation is given in Fig. 4.3.5. This circuit incorporates the necessary equalization.

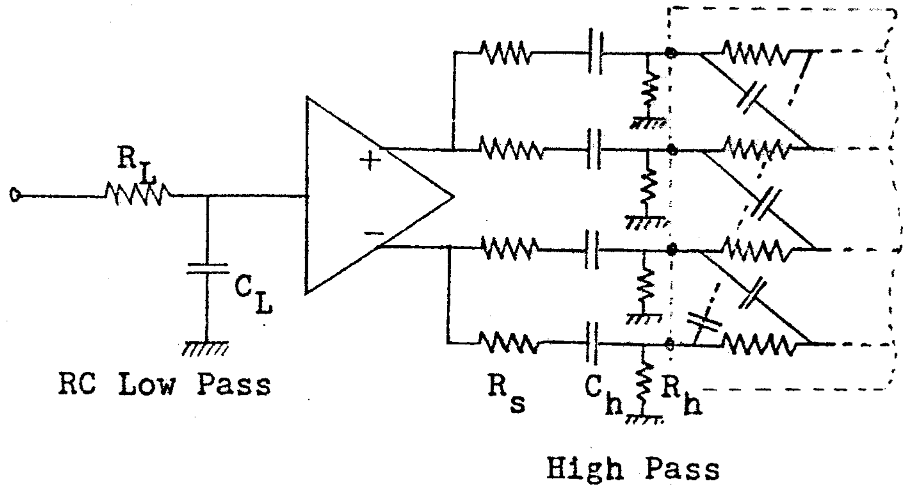
Because the equalization affects both positive and negative frequencies equally the resulting overall frequency response is very close to the theoretical shape except that the overall curve is lifted up by about 10.2dB. The ripple in the passband is higher at about 0.2dB but this is quite acceptable for many purposes and could easily be reduced with a little more sophistication in the equalising.

It is interesting to note that an identical frequency response, before equalization, can be obtained by reversing the order of the sections and making the resistors of all sections have the same value. The design values for this and several other types are tabulated in Figure 4.3.6. All can be equalised with simple passive R-C circuits in the manner described.

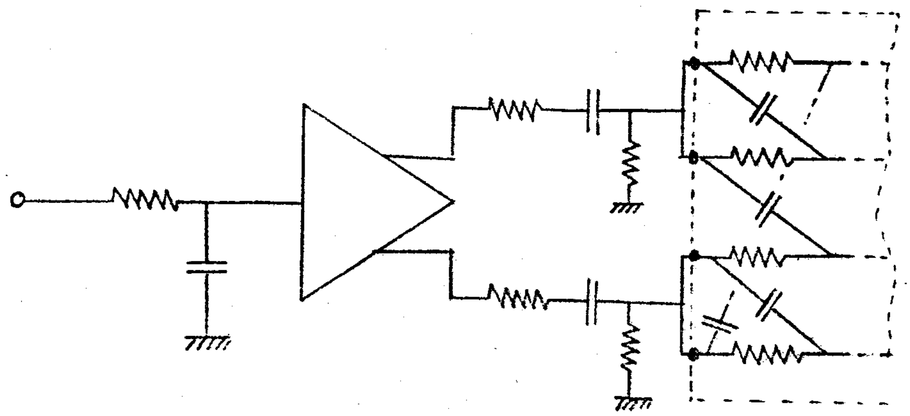
Filter (e) tabulated in Fig. 4.3.6 is of particular interest since the sections are arranged in an order more optimum for sensitivity than component value. This aspect will be discussed more fully in section 4.4.

4.3.4 Complete N-Phase Filters

Although the design principles can be extended to filters with other than 4 phases this has not been studied in any depth. Filters with 4 phases tend to be most economic in components. Even a three phase filter requires one more element per section than its four phase counterpart.

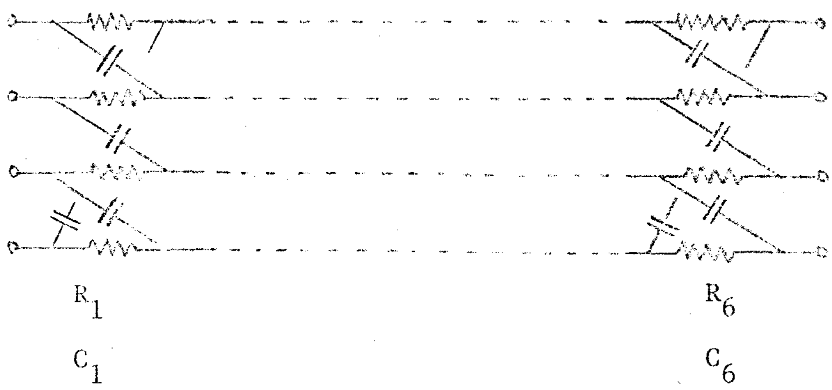


(a) Complete Drive Circuit.



(b) Simplified near equivalent.

Figure 4.3.5 Practical drive arrangements incorporating equalization of polyphase filters.



Six sections giving 50dB discrimination from -162.5Hz to -4000Hz.

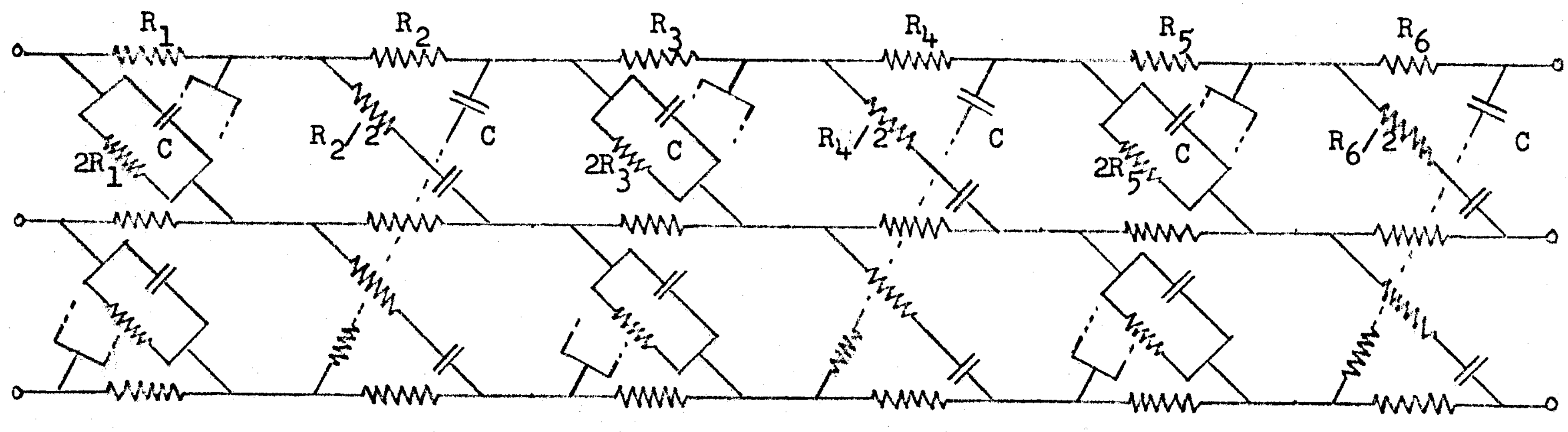
Figure 4.3.6 Four phase 6 section designs with component values.

	1	2	3	4	5	6
	(a) Ideal Optimised Design. Voltage source drives into no load.					
R kΩ C nF f kHz	1.0 42.733 3.7244	2.6929 25.602 2.3085	10.239 13.385 1.1610	43.097 6.6081 0.55885	132.86 4.2610 0.28113	401.25 2.2762 0.17426
	(b) Equal Capacitor Design. Equaliser H.P: .19μF, 6.8kΩ LP:CR=228.10^{-7} Rs=1kΩ					
R kΩ C nF f kHz	4.273 all as (a)	6.894 10nF	13.705	28.479	56.612	91.334
	(c) Equal Resistor Design.					
R kΩ C nF f kHz	all 91.334 0.17426	10kΩ 56.612 0.28113	28.479 0.55885	13.705 1.1610	6.894 2.3085	4.273 3.7244
	(d) Minimum Spread of Component Values.					
R kΩ C nF f kHz	30.221 30.221 .17426	23.794 23.794 0.28113	16.876 16.876 0.55885	11.707 11.707 1.1610	8.302 8.302 2.3085	6.537 6.537 3.7244
	(e) Sections arranged in order to improve sensitivity. 100μF/1MΩ/CR=2.5.10^{-5}/Rs=0					
R Ω C nF f kHz	42.734 1000 3.724	276.77 330 .17426	1.3708k 100 1.1610	8.630k 33 .55885	6.894k 10 2.3085	171.55k 3.3 0.28113

Some interesting theoretical studies were made of three phase filters. The types of sections A to D shown already in Figure 4.2.3 were considered. It was found that if a filter was designed using equal capacitors and sections all of one type then a very bad passband response was obtained. For example with all type A sections a slope of about 30dB across the passband was observed. It would be unrealistic to attempt to equalize this. This effect can be explained by considering the structure of a type A section. At zero frequency each successive section in the filter acts as a straight through connection and providing no final load is imposed then a unity gain will be found. At infinite frequency the capacitors shunt the skew resistors across each phase and a cascade of sections acts as a ladder attenuator.

With type C sections a similar situation arises but with high loss at zero frequency falling to unity gain at infinite frequency. The solution presented itself therefore of alternately using type A and C sections down the filter. A six section 3 phase filter designed on this principle and to the same 50dB specification as the previous 4 phase examples gave a response flat to within about 0.15dB from 200Hz to 4kHz without equalization. The absolute loss was 23.7 dB however and this would be generally undesirable. The filter schematic and component values are given in Figure 4.3.7 and the frequency response is plotted in Figure 4.3.8.

It was found that by modifying the impedance levels of the sections so that the capacitors in successive sections were in the ratio 32 : 16 : 8 : 4 : 2 : 1 that the overall



R k	3.701	7.961	11.87	32.88	49.03	105.46
F kHz	3.7244	2.3085	1.161	0.5589	0.2811	0.1743

All capacitors 10 nF

Figure 4.3.7 Three phase Equal capacitor 50 dB. Sequence asymmetric filter.

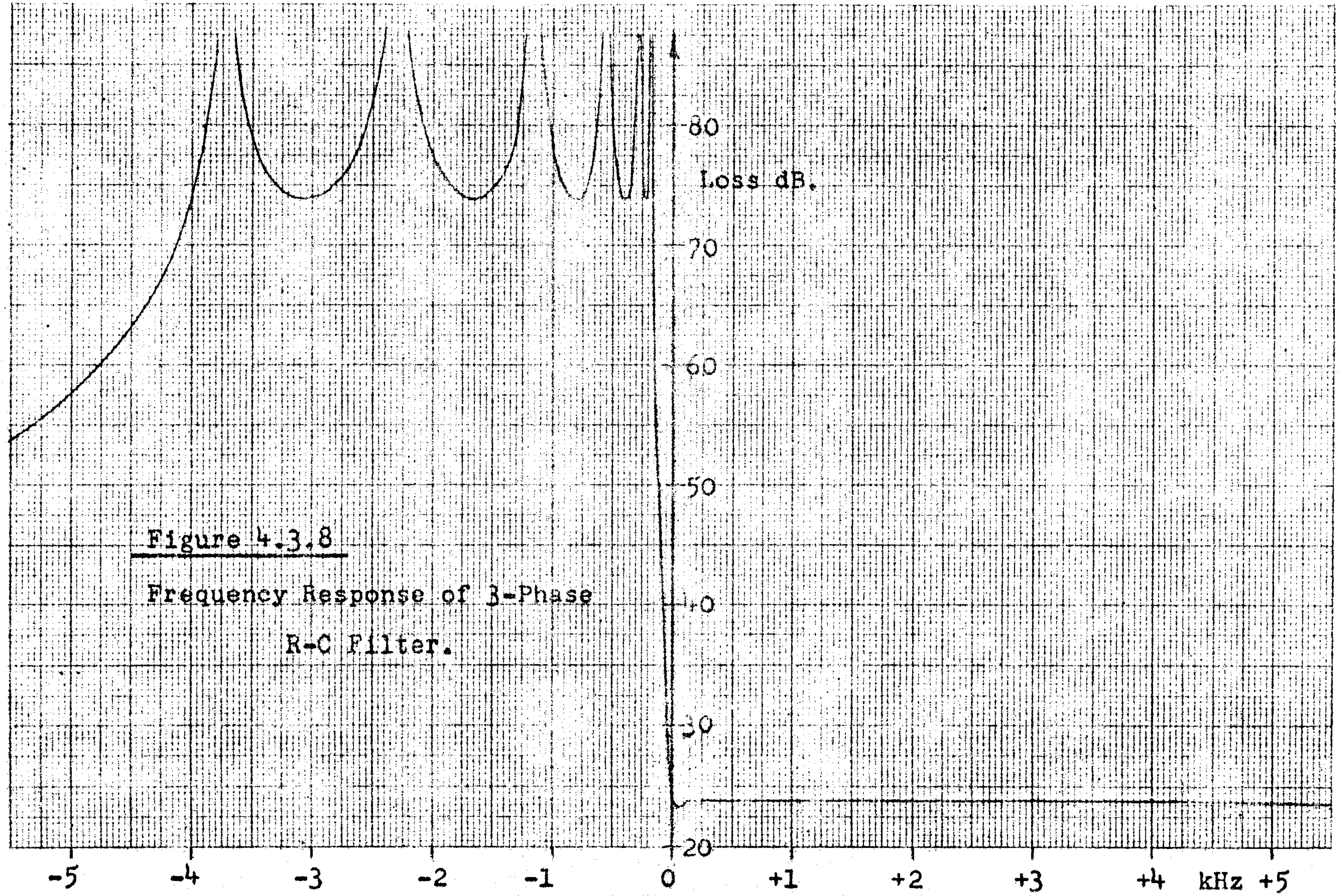


Figure 4.3.8

Frequency Response of 3-Phase R-C Filter.

loss was reduced by 8dB without significantly altering the shape of the frequency response.

Although no work has been done it can be expected that a similar approach with 5 or more phases should be successful. An important aspect to consider is the response of the filter to the N different symmetrical components that an input signal may be composed of. In most single sideband applications only the two main true N phase components are significant. In some other application this might not be the case and then the filter design would have to take into account the desired rejection characteristics for each unwanted symmetrical component.

4.4 Sensitivity of Lossy Asymmetric Filters

One of the most useful area of application of R-C asymmetric filters is to single sideband modulation and in this connection the sensitivity of the network response to variations in component values is very important. A network which requires all its components to have a tolerance of 0.1% will be much more expensive than if all or some of the tolerances can be relaxed to say 1%. To this end the sensitivity of 4 phase filters when used for single sideband modulation has been investigated in some depth. For comparison the performance of conventional quadrature modulation (Fig. 1.1.2) was used with two different designs of phase shift network. The quadrature modulation circuits were designed to give the same frequency response as the polyphase circuit and then comparisons were made of individual component sensitivity and using Monte Carlo analysis.

It is important to distinguish two kinds of sensitivity in a polyphase filter. Consider a 4 phase filter section such as the one shown in Fig. 4.1.1. If all the resistors move by +1% then the transmission zero moves by 1% without significantly altering the general shape of the response. This can be called 'Absolute' sensitivity. If however just one of the resistors moves +1% relative to the others then the network becomes unbalanced and the attenuation peak becomes rounded and finite. This will be called 'Relative Sensitivity'. It is this Relative Sensitivity which is the most critical factor in practical designs.

For comparative purposes the design used for most of the sensitivity analyses is the 50dB 6th order filter discussed in section 4.3. The nominal frequency response was given in figure 4.3.2 and the component values in figure 4.3.3. An additional design used for comparison with the 50dB design was a 70dB 6th order filter.

4.4.1 Relative Sensitivity

Consider the six section four phase design of Fig. 4.3.3. If all the components had exactly the right values the frequency response would be as shown in Fig. 4.3.2. with a minimum stopband discrimination of 50dB. Suppose now that only the first five sections have the correct values and that the last section has values as shown in Fig. 4.4.1(a). To make explanation of the imperfections easier suppose further that the network is driven as shown in Fig. 4.4.1(b) for the purpose of single sideband modulation and that the modulators do not load the network output in any way.

The action of this circuit has been explained in Chapter I and it can be seen that if the input signal is a

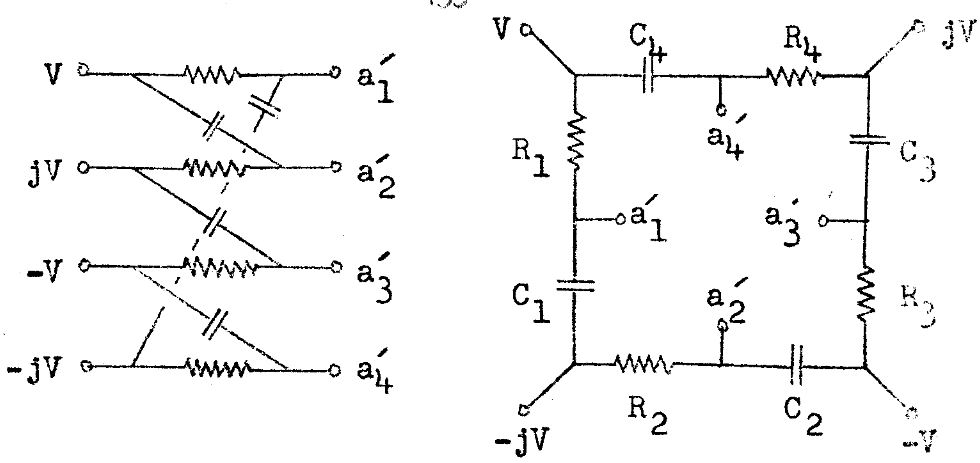


Fig. 4.4.1 (a)

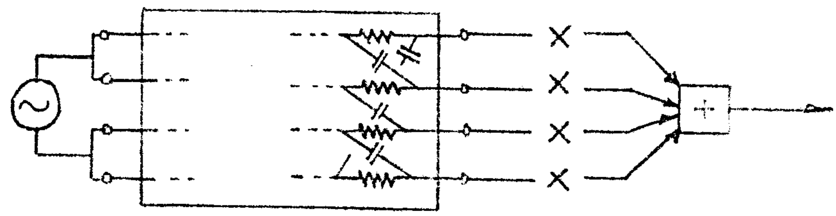


Fig. 4.4.1 (b)

Figure 4.4.1 Final section of a filter used in a single sideband modulator.

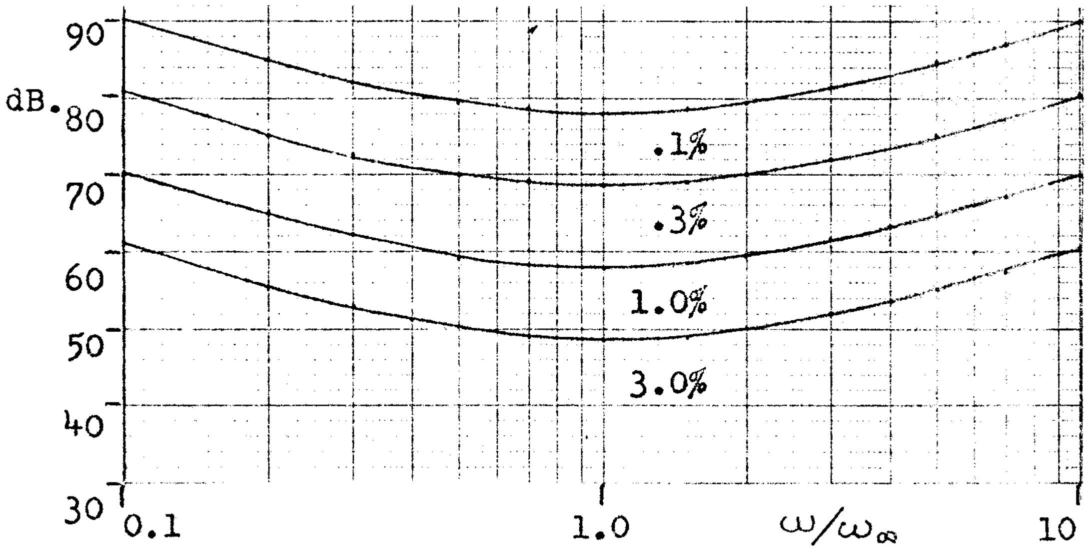


Figure 4.4.2 Sideband discrimination for various errors in the CR product of one phase of the final filter section.

sine wave at a frequency corresponding to one of the resonant frequencies of the first five sections then the signal at the sixth section input will be in near perfect 4 phase. Assuming, for simplicity of analysis that the phasing is perfect then:

$$a_1' = \frac{V(1+\omega C_1 R_1)}{1+j\omega C_1 R_1} = \frac{V(\omega_1+\omega)}{\omega_1+j\omega}$$

$$a_2' = \frac{jV(\omega_2+\omega)}{\omega_2+ j\omega}$$

$$a_3' = \frac{-V(\omega_3+\omega)}{\omega_3+j\omega}$$

$$a_4' = \frac{-jV(\omega_4+\omega)}{\omega_4+j\omega}$$

where $\omega_x = 1/C_x R_x$

Now the output can be resolved into symmetrical components as described in Chapter I. The symmetrical components are given by:

$$a_o = \tfrac{1}{4}\left[a_1'+a_2'+a_3'+a_4'\right]$$

$$a_1 = \tfrac{1}{4}\left[a_1'+ja_2'-a_3'-ja_4'\right]$$

$$a_2 = \tfrac{1}{4}\left[a_1'-a_2'+a_3'-a_4'\right]$$

$$a_3 = \tfrac{1}{4}\left[a_1'-ja_2'-a_3'+ja_4'\right]$$

At the input to the 6th section $a_o = a_1 = a_2 = 0$ and $a_3 = V$ (perfect phasing assumed). This is not so at the output and when applied to the modulators the a_1 component, now no longer zero, gives rise to the unwanted sideband. By substituting the output voltages a' into the symmetrical component formulae the ratio of wanted to unwanted sideband discrimination can be found.

$$\frac{a_3}{a_1} = \frac{a_1'-ja_2'-a_3'+ja_4'}{a_1'+ja_2'-a_3'-ja_4'}$$

It should be noted that components a_o and a_2 do not give rise to any output with a perfect set of modulators.

Consider first the case where just one resistor or capacitor has an incorrect value so that

$$\omega_1=\omega_o+\delta \qquad \omega_2=\omega_3=\omega_4=\omega_0$$

Then

$$a_1' = \frac{V(\omega_o+\delta+\omega)}{\omega_o+\delta+j\omega}$$

and $$\frac{a_2'}{j} = \frac{a_3'}{-1} = \frac{a_4'}{-j} = \frac{V(\omega_o+\omega)}{\omega_o+j\omega}$$

So $$\frac{a_3}{a_1} = \frac{\left[\dfrac{\omega_o+\delta+\omega}{\omega_o+\delta+j\omega} + 3\,\dfrac{\omega_o+\omega}{\omega_o+j\omega}\right]}{\left[\dfrac{\omega_o+\delta+\omega}{\omega_o+\delta+j\omega} - \dfrac{\omega_o+\omega}{\omega_o+j\omega}\right]}$$

which can be re-arranged as

$$\frac{a_3}{a_1} = \frac{4(1+x) + y + 3y\ (1+x) + jx\ (4+4x+y)}{xy\ (j-1)} \sim \frac{4(1+x)(1+jx)}{xy(j-1)}$$

where $x = \frac{\omega}{\omega_o}$ and $y = \frac{\delta}{\omega_o} << 1$

Single sideband discrimination $20 \log_{10} \left|\frac{a_3}{a_1}\right|$ dB is plotted in Fig. 4.4.2 for a range of errors in C_1R_1 from 0.1% to 3%. This is independant of any particular design but shows what is likely to happen for input frequencies corresponding to the resonant frequency of any of the sections preceding the final section. The foregoing analysis is in fact an approximation because it assumes that the input signal to the final section is in exact 4 phase. In practise this is only true at the frequency corresponding to the penultimate section. The slight asymmetry of the last section causes slightly unequal currents to be drawn in each phase which causes unequal voltage drops through intervening sections. For example, if the input frequency corresponds to the resonant frequency of the first section of the filter then the output from that section will be a perfect 4 phase signal. Due to the unequal loading by the last section the voltage drops in each phase will not be exactly the same so that by the time the signal reaches the last section it will no longer be in perfect 4 phase. This is well illustrated in Fig. 4.4.3 which takes the 6th order 50dB design of Fig. 4.3.3 as an example. Curve (a) shows the theoretical response of the stopband (ie. the sideband discrimination when used as a modulator). Curves (b) and (c) show the result of a 3% error in one capacitor or

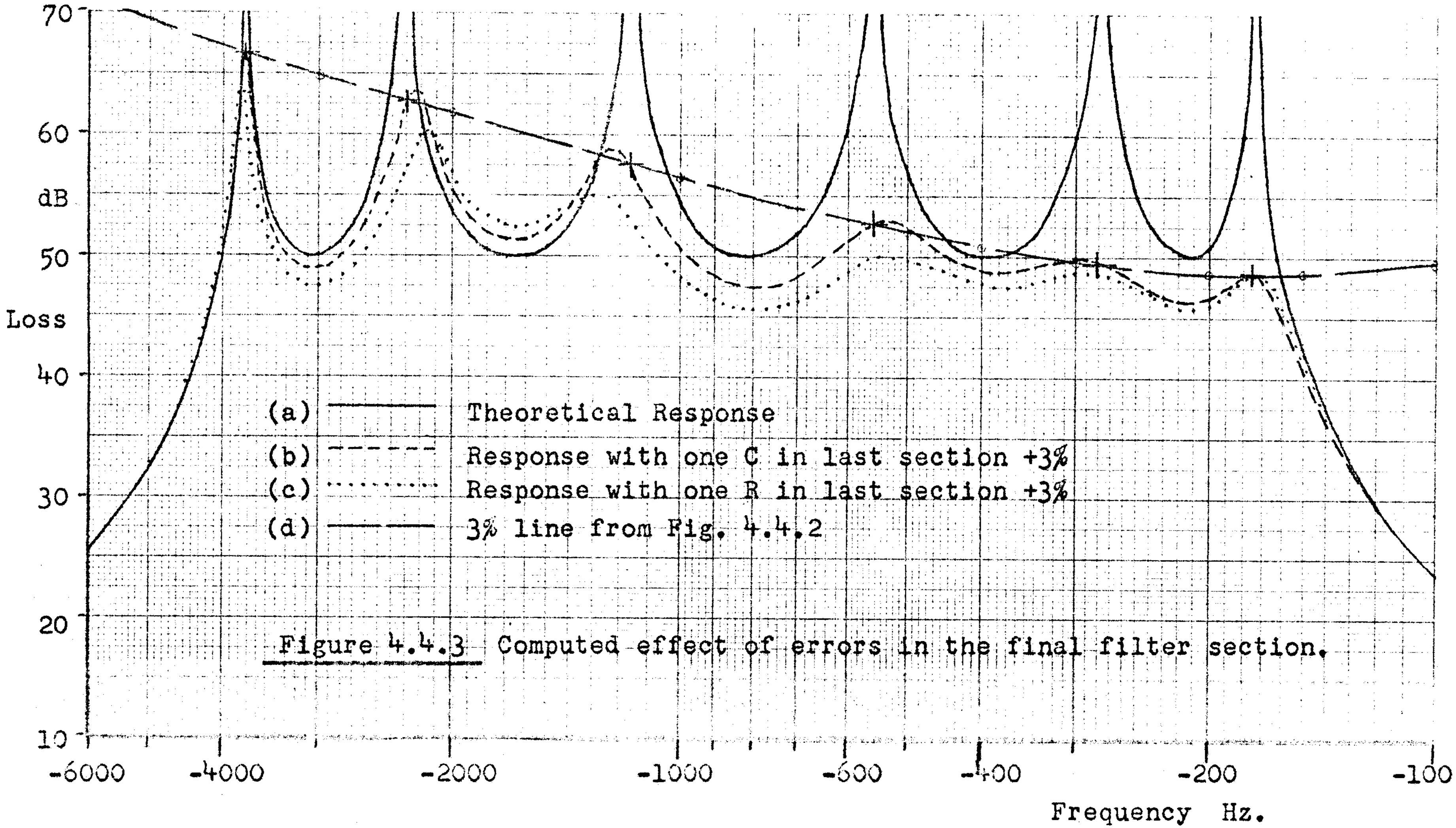


Figure 4.4.3 Computed effect of errors in the final filter section.

resistor in the final section respectively. These were evaluated using a computer nodal analysis programme. Curve (d) is the 3% curve taken from Fig. 4.4.2 and it can be seen that while the capacitor response intersects the expected curve almost exactly at the transmission zero frequencies the resistor curve does not. This is because in this particular design a resistor error causes a much more significant unbalanced loading at higher frequencies corresponding to the earlier sections than a capacitor error. This position would be reversed if the order of the sections was to be reversed.

At frequencies in between the transmission zeros the response is almost always worse than the theoretical response the minimum attenuation being 45.5dB in this example. Also the capacitor error case is nearly always better than the resistor error case. This could be important in deciding the preferred order of sections if close tolerance capacitors proved to be more costly than close tolerance resistors.

The approximate theory so far developed can be extended in the following way to cover the case where all components in the final section depart from nominal simultaneously.

Let $$\frac{a_1'}{V} = \frac{\omega_1 + \omega}{\omega_1 + j\omega} = \frac{1+y_1+x}{1+y_1+jx}$$

$$\frac{a_2'}{V} = j\left[\frac{\omega_2+\omega}{\omega_2+j\omega}\right] = j\frac{1+y_2+x}{1+y_2+jx}$$

$$\frac{a_3'}{V} = -\left[\frac{\omega_3+\omega}{\omega_3+j\omega}\right] = -\frac{1+y_3+x}{1+y_3+jx}$$

$$\frac{a_4'}{V} = -j\left[\frac{\omega_4+\omega}{\omega_4+j\omega}\right] = -j\,\frac{1+y_4+x}{1+y_4+jx}$$

where $\omega_r = 1/C_rR_r = \omega_o + \delta_r$

$$x = \frac{\omega}{\omega_o}$$

$$y_r = \frac{\delta r}{\omega_o}$$

Therefore the unwanted sequence output is

$$\frac{4a_1}{V} = \frac{1+y_1+x}{1+y_1+jx} - \frac{1+y_2+x}{1+y_2+jx} + \frac{1+y_3+x}{1+y_3+jx} - \frac{1+y_4+x}{1+y_4+jx}$$

$$\sim \frac{x(j-1)(y_1-y_2+y_3-y_4)}{(1+jx)^2}$$

using the binomial theorem and assuming all y_r very small. Similarly the wanted output is

$$\frac{4a_3}{V} \sim \frac{4(1+x)}{1+jx}$$

and discrimination is given by

$$\frac{a_3}{a_1} \sim \frac{4(1+x)(1+jx)}{x(y_1-y_2+y_3-y_4)(j-1)}$$

This analysis while approximate is very useful in clearly showing the worst case situation which would exist when $y_2=-y_1, y_3=y_1, y_4=-y_1$. It also suggests that the critical point in the design is to match C_1R_1 and C_2R_2 very closely together and to match C_3R_3 with C_4R_4. This in turn indicates that it should be possible to adjust the section for near optimum results with two controls say on R_1 and R_3.

Towards the input end of the filter the sections are much more tolerant of relative tolerance errors. This will be illustrated with the same 6th order design example. If only the first section is in error and all the remaining sections are perfect then no matter what happens in the first section subsequent sections will still give their theoretical contributions to the overall response. This is illustrated in Fig. 4.4.4 where the first section has been removed entirely. It can be seen from this that a change in the contribution from the first section will not make very much difference to the overall response particularly at lower frequencies. This is supported by curve (c) which shows the computed response for the case where one resistor is 10% low in the first section. The main changes occur at frequencies close to the resonant frequency of the first section where the minima has moved by about 1.5dB. At the low frequency end of the stopband the change is so small as to be undetectable on the scale of the figure.

Relative sensitivity is therefore much more critical for sections at the output of the filter. Experiment has shown that intermediate positions give intermediate results and that the sensitivity is related to the section position and not the order or sequency of sections. This is supported by the theoretical analysis given.

Finally some figures can be given derived by substituting into the formula for the worst case situation in the last section.

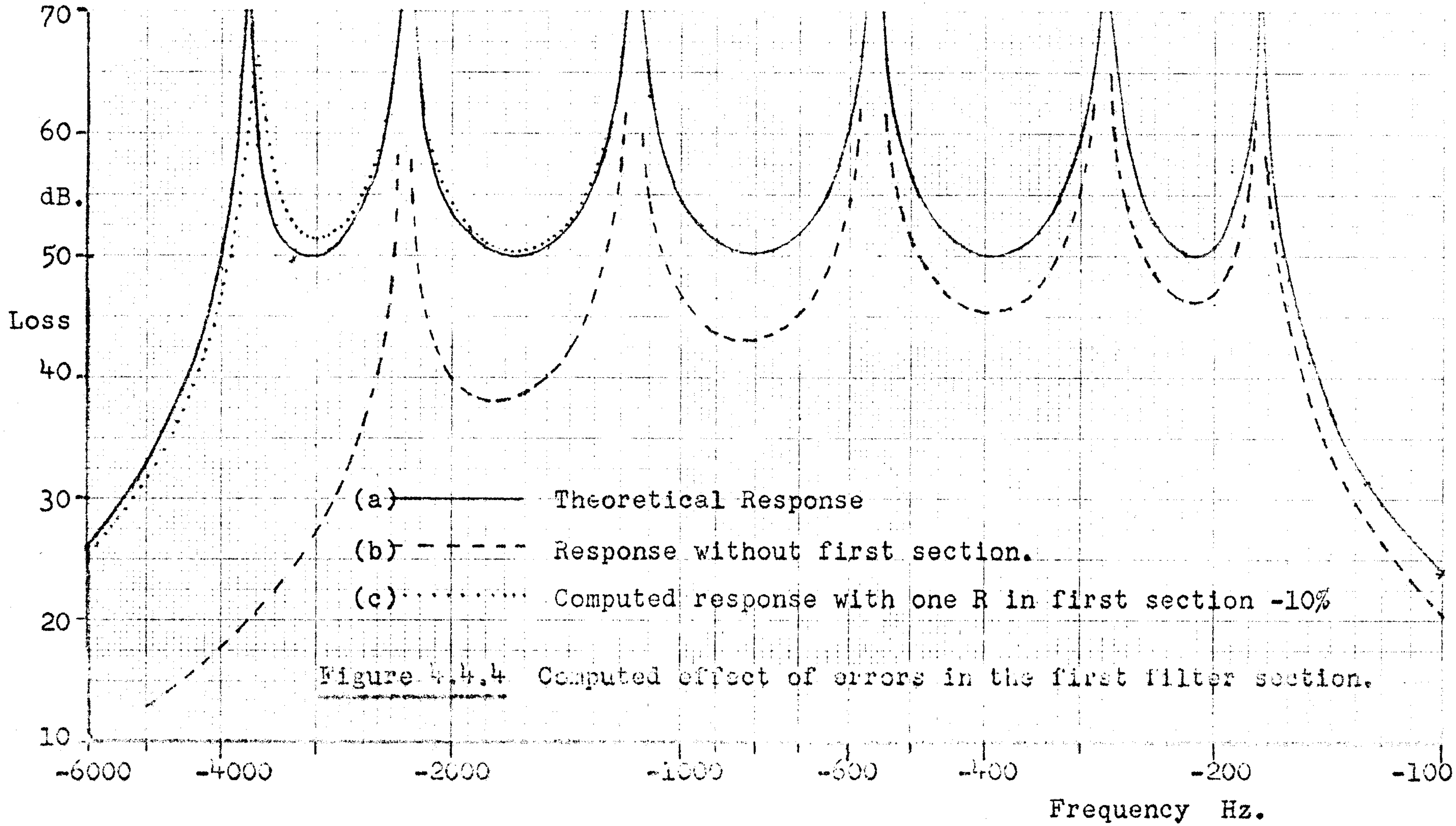


Figure 4.4.4 Computed effect of errors in the first filter section.

Tolerance %	Attenuation at last section frequency
3	36.5 dB
1	46 dB
0.3	56.5 dB
0.1	66 dB

4.4.2 Absolute Sensitivity

The condition where all the resistors (or capacitors) move together by the same amount from nominal has been defined as absolute sensitivity. In this situation the filter still remains physically symmetrical and the transfer function is modified only slightly by the small movement of the transmission poles and one transmission zero.

A 3% change in all the resistors of one section causes a 3% movement in the transmission zero frequency associated with that section. This causes the stopband minimum on one side of the zero to be reduced and that on the other side to be increased in attenuation. As an example the 6th order 50dB design of Fig. 4.3.3 was analysed to find the effects of 3% absolute tolerances on all the components of each section. In the stopband the minima were found to be reduced by 1.1dB in the worst case so that the overall sideband discrimination would not fall to less than 48.9dB if one section had all its resistors or capacitors out by 3%. In the passband no change of more than .05dB was found and this is representative of the fact that these filters are extremely insensitive in the passband.

The absolute sensitivity of these filters has much in common with the sensitivity characteristics of the single phase lattice filter which is very sensitive in the stopband and very insensitive in the passband. The reasons for this are similar because, like the lattice filter, the stopband is achieved by balancing branch currents against one another where as the passband is obtained by the addition of the branch currents.

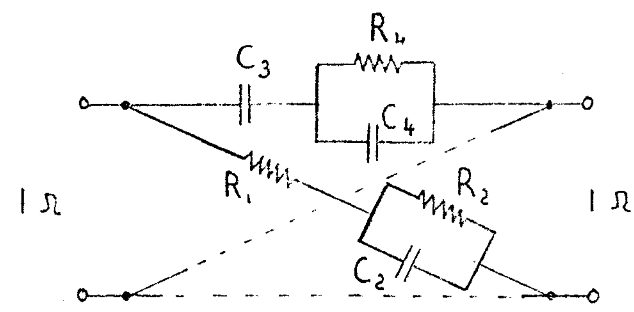
It is not useful to be specific as to what absolute tolerances are required in a given situation. It is generally better to consider each case on its merit. Absolute tolerance can always be made less critical by using a higher order design so that the initial attenuation is higher than required leaving a margin for absolute errors. Widening the stopband will also give an increased margin for error.

In practice none of the components in the filter will have their exact design values and the next section considers this through the use of Monte Carlo Analysis.

4.4.3 Sensitivity Comparisons with Other Methods

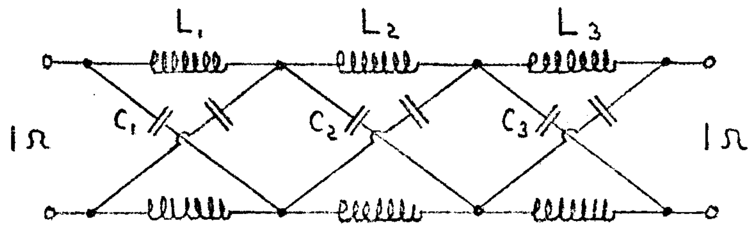
Practical filters must be built using components none of which will have their exact design values. Each component will, in general, have a random value within a band of values dependant on the tolerance of that component. Monte Carlo analysis is a useful computer technique of assessing the effect of random variation in component values. For each component the computer selects a value within the assigned tolerance range. When this has been done for every component the programme will assess the performance of the filter by circuit analysis. This whole procedure is then repeated many times so that a probability distribution of performance can be obtained. Component values may be selected on a random basis within the allowed tolerance band or according to some probability distribution given for each component. Network performance can be assessed on such criteria as passband ripple, stopband attenuation, phase, group delay or impulse response. in this instance the most critical parameter is stopband attenuation which determines the single sideband discrimination.

For comparison conventional quadrature modulation using phase splitting networks was considered. This has been briefly mentioned in Chapter 1 and shown in Figure 1.1.2. Two phase shifting all-pass networks are required for this method the networks being designed so that the phase difference between their outputs approximates to 90° over the desired transmission band. For the purpose of this comparison the networks were designed to give the same transfer function when used as a single sideband modulator as the equivalent passive R-C polyphase design. Two designs of phase splitter were used. The first was a passive



	50dB		70dB	
	ϕ_1	ϕ_2	ϕ_1	ϕ_2
R_1	.2111422	0.21119787	.1811727	.1811727
R_2	4.523675	4.523698	5.338408	5.3384105
C_2	.7017024	1.561292	.747889	1.429393
C_3	1.918644	6.006806	1.6261627	4.397077
R_4	.7904129	1.0551314	.6965387	1.0621848
C_4	.1822937	.5707300	.2644007	.7149285

a) Passive R-C lattice



	50dB		70dB	
	ϕ_1	ϕ_2	ϕ_1	ϕ_2
$L_1=C_1=$	.078981	.285692	.105546	.352427
$L_2=C_2=$	.670398	1.49164	.72334	1.382475
$L_3=C_3=$	3.50023	12.6614	2.837471	9.474556

b) Passive L-C lattice

Figure 4.4.5.

Phase Splitting Networks Used for Comparison with the Passive Polyphase Method.

R-C lattice design according to the principles of Weaver and others (Ref. C8). The second type was a passive constant impedance L-C lattice. Two transfer functions were considered, a 50dB 6th order design for comparison with and identical to the standard polyphase filter of Figures 4.3.2 and 4.3.6, and a similar 70dB 6th order design. The normalised design values for these phase splitting networks are given in Fig. 4.4.5. It is useful to note that the L-C design normalised values can be read directly from the tables of poles and zeros given in Appendix 1.

<u>Table of Individual Component Sensitivities</u>

<u>50dB 6th Order Design.</u>

<u>Maximum Degradation in dB for 1% Component Change.</u>

	Quadrature Modulation				Polyphase	
	Passive RC Fig 4.4.5(a)		Passive LC Fig 4.4.5(b)		Equal Capacitor Fig 4.3.6(b)	
ϕ_1	C_1	6.70	C_1	2.06	C_1	0.42
	R_1	6.01	L_1	2.06	R_1	0.44
	C_2	4.07	C_2	2.74	C_2	0.43
	R_2	2.85	L_2	2.74	R_2	0.39
	C_3	7.57	C_3	2.53	C_3	0.69
	R_3	4.09	L_3	2.53	R_3	0.60
ϕ_2	C_1	2.80	C_1	2.82	C_4	0.54
	R_1	5.83	L_1	2.82	R_4	0.66
	C_2	6.42	C_2	2.69	C_5	0.72
	R_2	4.25	L_2	2.69	R_5	0.81
	C_3	7.33	C_3	1.69	C_6	0.86
	R_3	2.40	L_3	1.69	R_6	1.54
	Source and Load Resistors				Negligible sensitivity to source resistor and equaliser components. (< .02 dB)	
ϕ_1	R_S	4.47	R_S	2.19		
	R_L	4.70	R_L	2.17		
ϕ_2	R_S	5.13	R_S	2.19		
	R_L	4.56	R_L	2.17		

Before proceeding to Monte Carlo analysis a comparison was made of the individual component sensitivities of the 50dB design. The criterion of sensitivity in this case was taken to be the maximum change for the worse in the sideband discrimination below the theoretical 50dB. The results are tabulated on the previous page for a 1% change in any single component. It can be seen that the RC phase splitting network is by far the worse exhibiting degradations of up to 7.5dB whereas the polyphase design of Fig. 4.3.6(b) is the best having a maximum degradation of 1.5dB. The LC phase splitter falls between these two being at worst 2.8dB. Further, the polyphase design is most sensitive at one end only compared with the other designs where most of the components are equally sensitive.

These same networks were then compared using a Monte Carlo analysis program with all component values selected at random from a range about nominal of ±1% This was done for 100 runs on each network classifying the result according to the minimum stopband attenuation achieved for each run. The results plotted as probability histograms are given in Fig. 4.4.6 where the same performance as was given by individual component sensitivities is repeated. In this case 95% of the polyphase networks had discriminations in excess of 45dB compared with 40% for the passive LC phase splitter and 8% for the RC phase splitter.

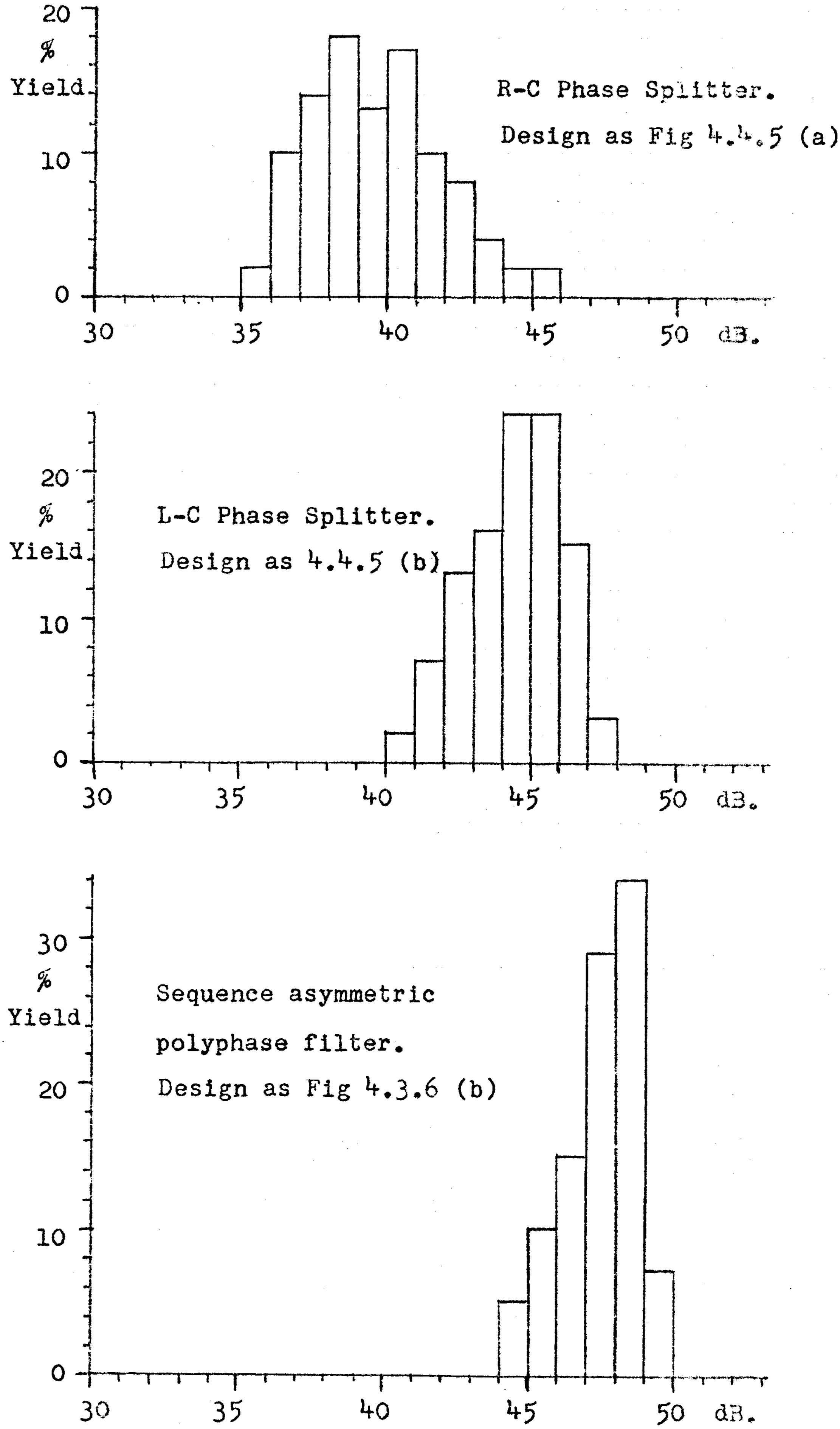


Figure 4.4.6 6th order 50 dB designs with 1% components. Histograms of computed minimum sideband discrimination. From 100 run Monte Carlo.

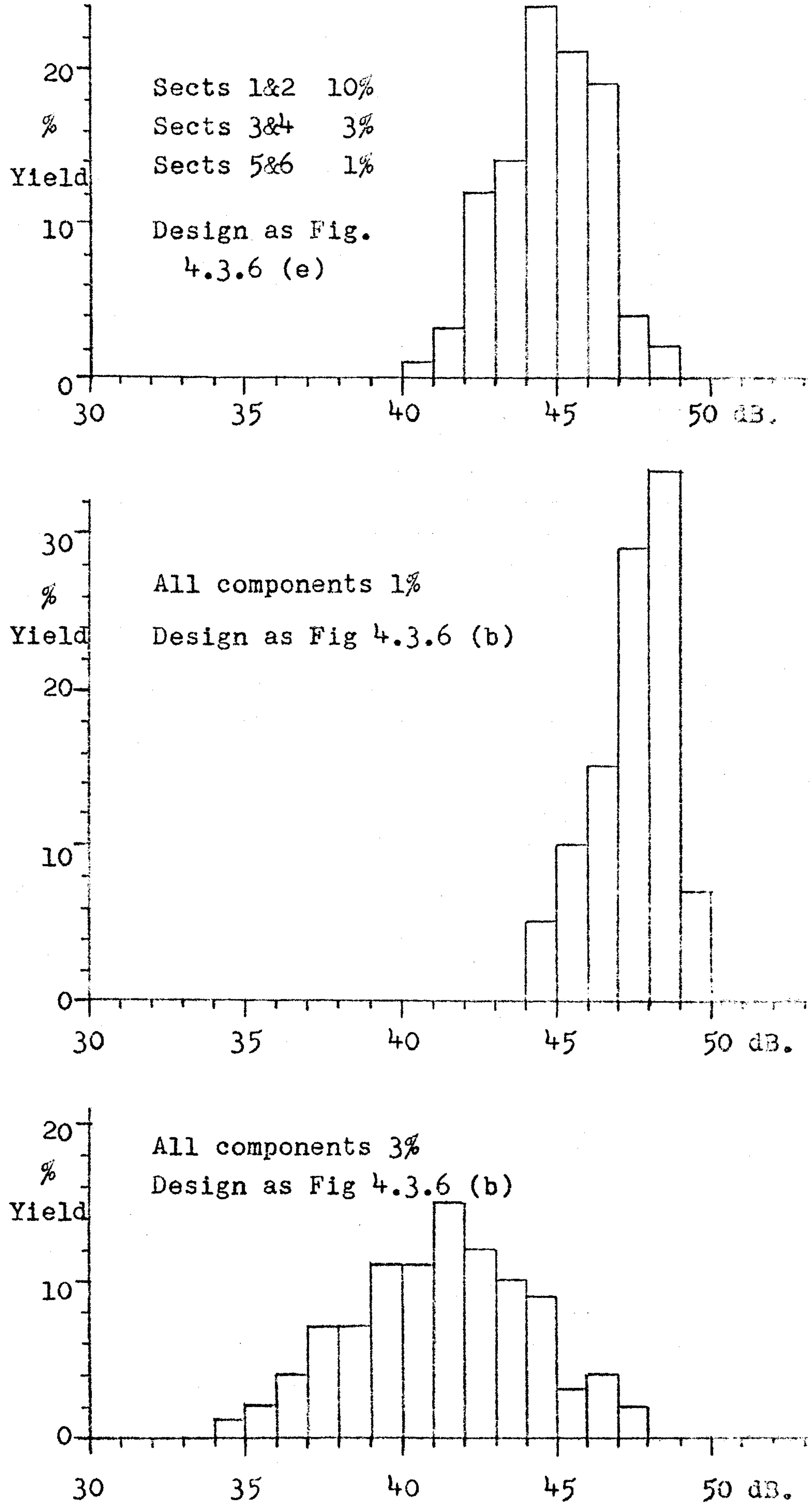


Figure 4.4.7 6th order 50 dB polyphase filter.
Histograms of minimum sideband
discrimination. From 100 Monte Carlo runs.

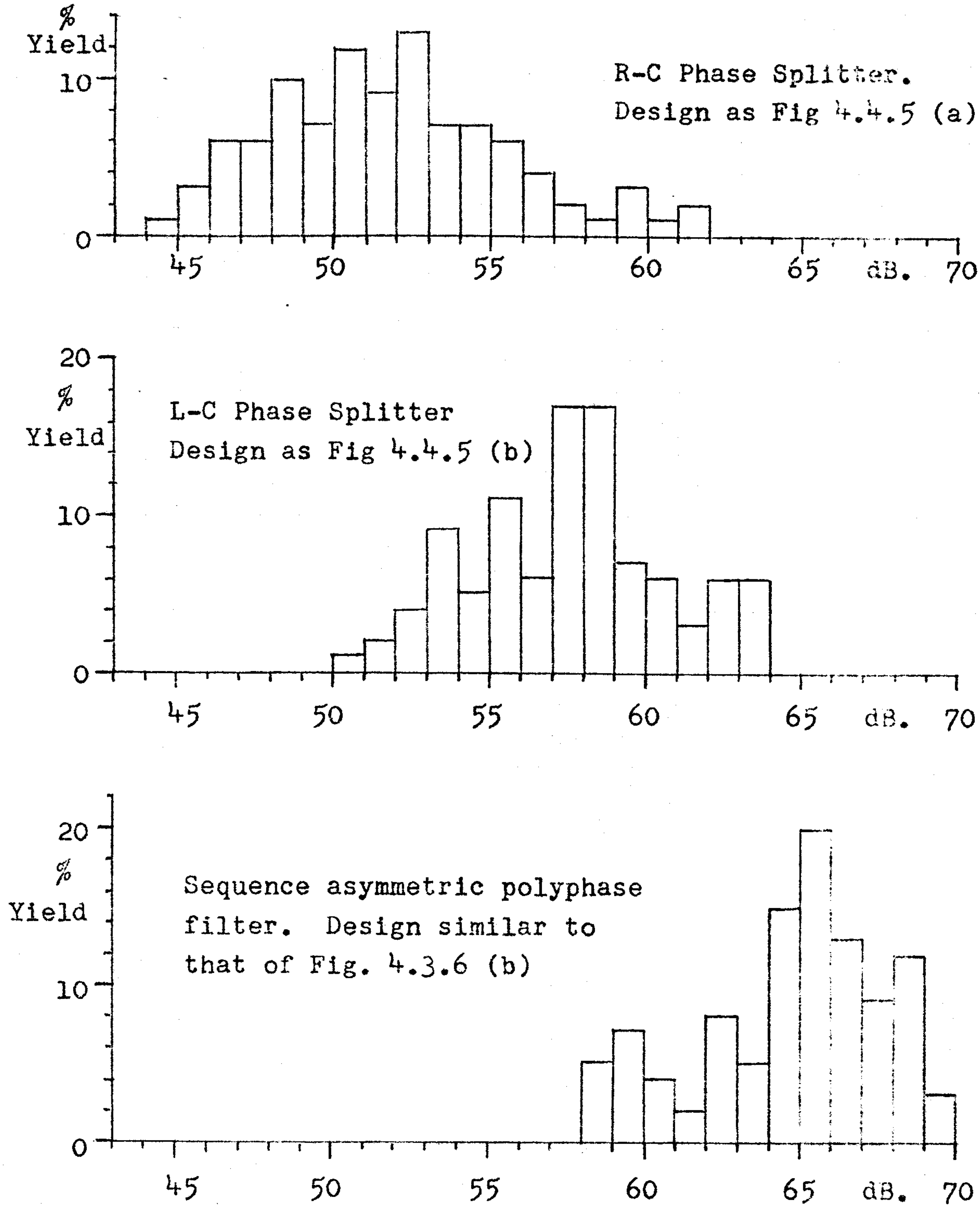


Figure 4.4.8 6th order 70 dB designs with .3% components. Histograms of computed minimum sideband discrimination. From 100 Monte Carlo runs. Rectangular tolerance distribution.

Some further investigations were made of the polyphase design alone for different tolerances and for grading the tolerances along the network. The results of this are given in Fig. 4.4.7 where it can be seen that grading the tolerances from 10% at the input to 1% at the output section gives considerably superior performance to making all components 3%. In order to take advantage of the grading it appears from experiment to be advantageous to choose the order of the sections to get maximum coverage of the stopband from each tolerance. This was done and the design appears in Fig. 4.3.6(e) the results of Figure 4.4.7 being obtained using this design.

A 70dB design was also compared with the phase splitting method and the performance histograms for 0.3% component tolerances are given in Fig. 4.4.8. In general these results are similar to those obtained with the 50dB design except that the distributions are more spread out.

An attempt was made, based on Monte Carlo analysis to produce curves indicating the expected performance of single sideband modulation networks for various tolerances. This work was limited by computer time to considering just the 50dB and 70dB designs already discussed. As a result some approximate curves are given in 4.4.9 showing the minimum discrimination achieved by 90% of networks made with a given component tolerance. While these curves are only approximate they do give useful information. For example whereas it would be advantageous to increase the accuracy of components in a 50dB RC phase splitter design from 0.3% to 0.1% it would be

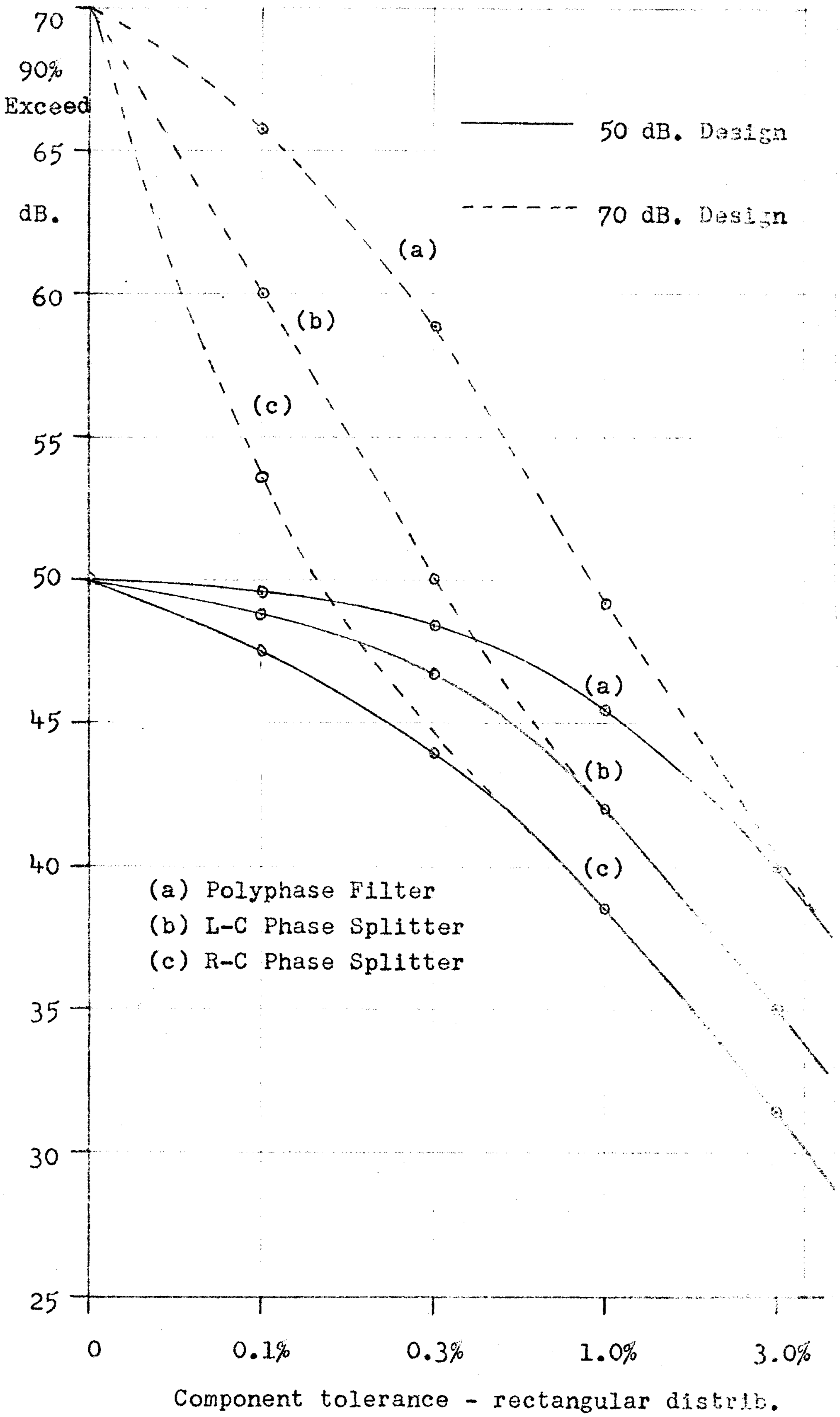


Figure 4.4.9 Probability of 90% of networks acheiving a minimum performance with given component tolerances.

unnecessary and unprofitable in a polyphase design. Also indicated is that by initially designing a network with a higher discrimination than actually required then less expensive wider tolerance components can be used to still meet the specification. This is to some extent making use of the statistics of the situation where using more components increases the probability that the average deviation will be zero. This is also one reason for the high yield of the 4 phase RC polyphase design in that it uses twice as many elements as the phase splitting equivalents. The other, more important, reason is that it is fundamentally less sensitive.

4.5 Summary of Lossy Polyphase Networks

A broad class of RC passive sequence asymmetric polyphase networks has been presented. These networks offer considerable advantages over equivalent known networks when used for phase splitting and single sideband modulation. These advantages reside principally in the easier component tolerances required and the much more practical component values that are possible.

Using the polyphase technique it is just as easy to split a signal into any integer sub-division of 360^{o} as to split the signal into two 90^{o} apart so that the general N phase filter can be usefully considered for such applications.

While the sensitivity of active phase splitting networks has not been included, in the comparisons of section 4.4.3, the best of such designs should be as good as the L-C phase splitter. An example (C13) uses thin film circuits for the 0.1% accurate RC products needed to provide 50dB discrimination.