CHAPTER 1

Introduction

1.1 Introductory Background

The problem of cheap and efficient single sideband modulation has attracted an immense amount of research and development over the last fifty years. The first single sideband modulators used direct amplitude modulation which produces both upper and lower sidebands one of which is selected by a bandpass filter. (Fig. 1.1.1). This is still the most popular method today although considerable effort has been spent in the search for alternatives. Many systems impose such stringent requirements on the unwanted sideband suppression that the bandpass filters become very expensive.

One of the best known alternatives which largely alleviates the filter selectivity problem is "Quadrature Modulation" (Fig. 1.1.2) which splits the signal to be modulated into two paths. By means of phase shift networks the signals in the two paths are arranged to be out of phase by approximately 90^{o} over a defined bandwidth. Application of each signal to its respective modulator, with carriers 90^{o} apart and adding the resultant modulation products results in controlled suppression of one sideband. There are a number of variants on this (B1-B5) but all suffer from the same drawback of high sensitivity to the tolerances of the components of the phase shift networks (as will later be demonstrated).

There is also the "Third Method" or "N-Path Filter" due to Barber, Weaver and others (D1, D5). This relies on double modulation and filtering (Fig. 1.1.3). The principle is similar

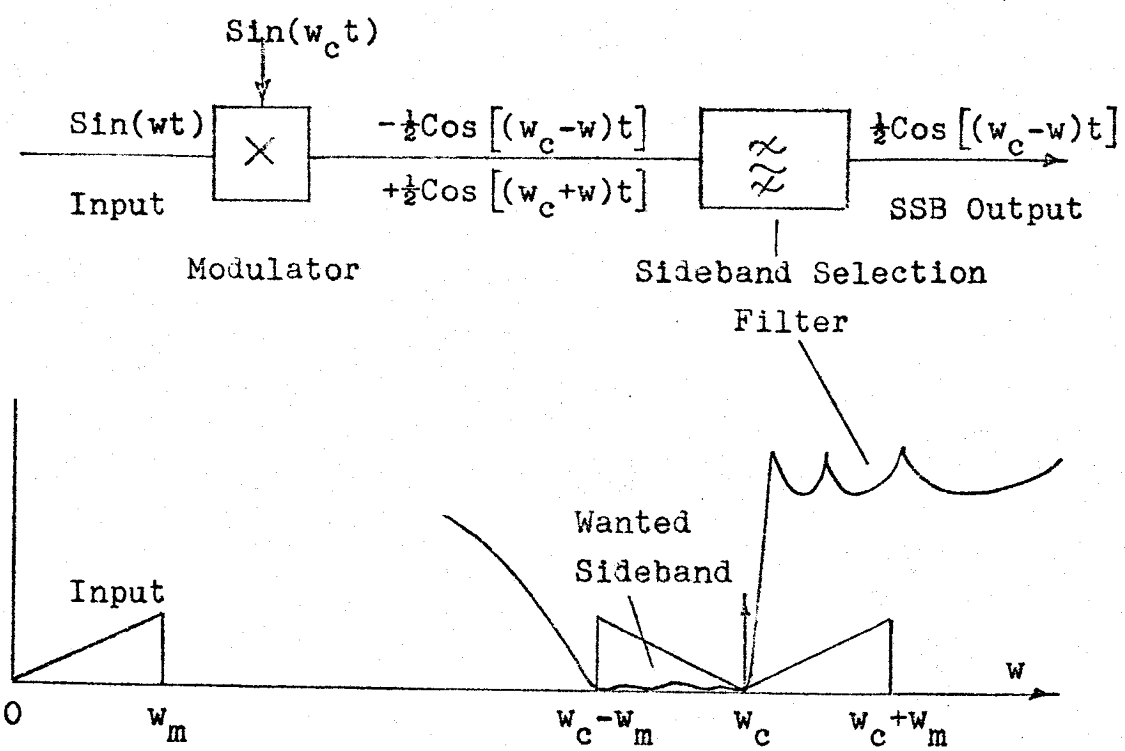


Figure 1.1.1 Conventional Single Sideband Modulation

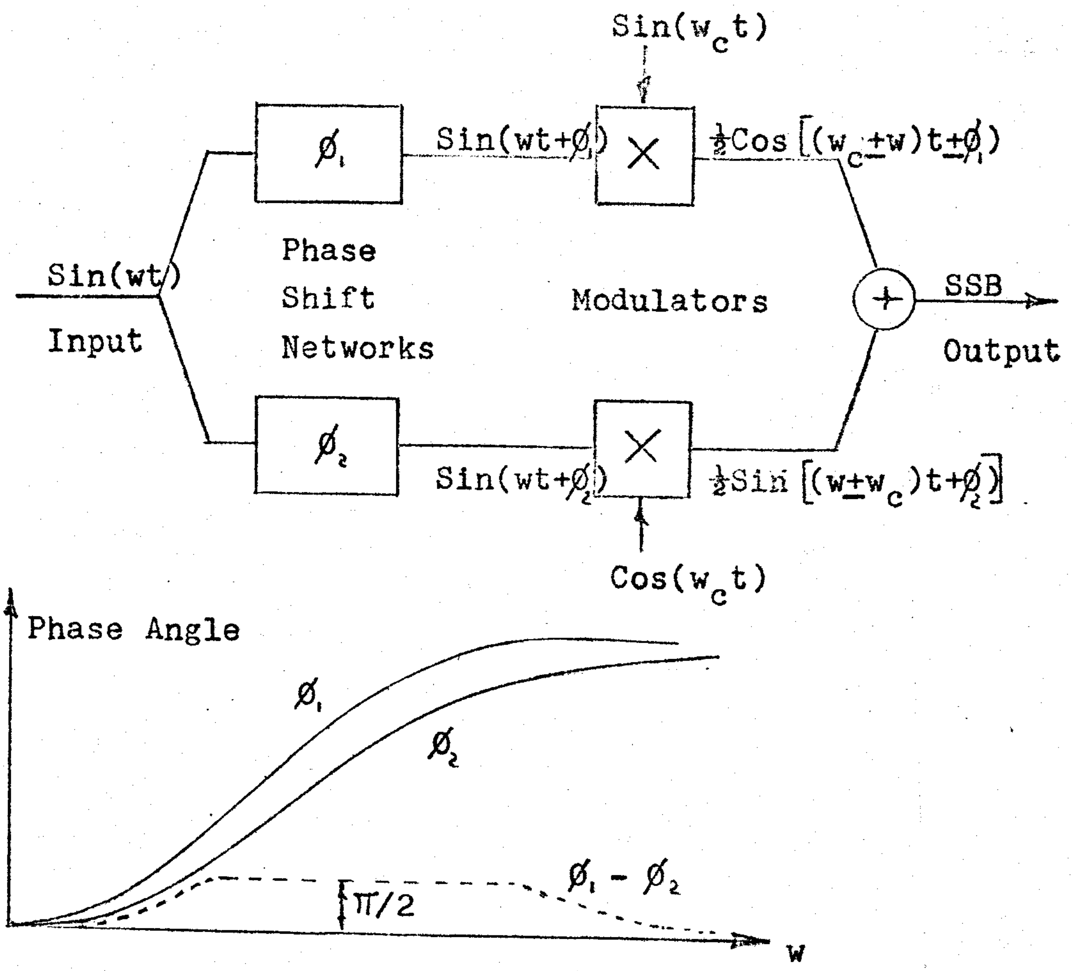
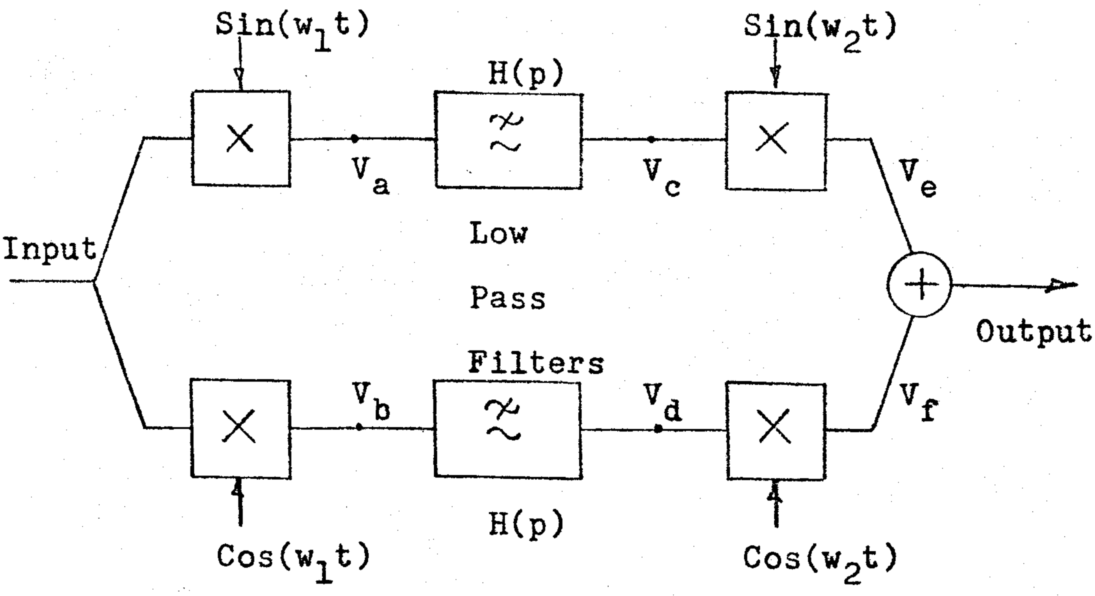


Figure 1.1.2 Quadrature Modulation.



If Input = A.Sin(wt)

Then:

$$V_a = \tfrac{1}{2}A\left[\mathrm{Cos}(w-w_1)t - \mathrm{Cos}(w+w_1)t\right]$$

$$V_b = \tfrac{1}{2}A\left[\mathrm{Sin}(w+w_1)t + \mathrm{Sin}(w-w_1)t\right]$$

$$V_c = \tfrac{1}{2}A.\mathrm{Cos}\left[(w-w_1)t+\emptyset\right] . \left|H(p-p_1)\right|$$

$$V_d = \tfrac{1}{2}A.\mathrm{Sin}\left[(w-w_1)t+\emptyset\right] . \left|H(p-p_1)\right|$$

$$V_e = \tfrac{1}{4}A\left[\mathrm{Sin}\overline{(w+w_2-w_1)t+\emptyset} + \mathrm{Sin}\overline{(w_1+w_2-w)t-\emptyset}\right] . \left|H(p-p_1)\right|$$

$$V_f = \tfrac{1}{4}A\left[\mathrm{Sin}\overline{(w+w_2-w_1)t+\emptyset} - \mathrm{Sin}\overline{(w_1+w_2-w)t-\emptyset}\right] . \left|H(p-p_1)\right|$$

$$\text{Output} = V_e + V_f$$

$$= \tfrac{1}{2}A.\left|H(p-p_1)\right|.\mathrm{Sin}\left[(w+w_2-w_1)t+\emptyset\right]$$

$\emptyset$= Low pass filter phase shift at $w-w_1$

<u>Figure 1.1.3</u> Weaver or 'N-Path' method for Single Sideband Modulation.

to quadrature modulation except that the initial 90° phase splitting of the signal is achieved by the first set of modulators. The modulators translate the low pass filter responses by a linear frequency shift producing an effective single sideband filter that is arithmetically symmetrical (refs. D1, D5, D6-D11). Because the modulators operate with carrier frequencies right in the center of the transmission band any carrier leak, due to modulator unbalance for example, appears as a steady tone at a frequency where it is most objectionable. The implementation of a high quality transmission system using this method therefore involves considerable practical difficulties.

After working for some time on N Path modulation and filtering techniques it became apparent that there should be some way of generating asymmetric filtering characteristics. For frequency division multiplex the requirements are usually highly asymmetric and the straight forward N Path modulator is not very efficient. An attempt was made to cause asymmetric distortion of a particular N Path modulator which used LC low pass filters in each path. This was done by skew connecting resistors between the paths as shown below.

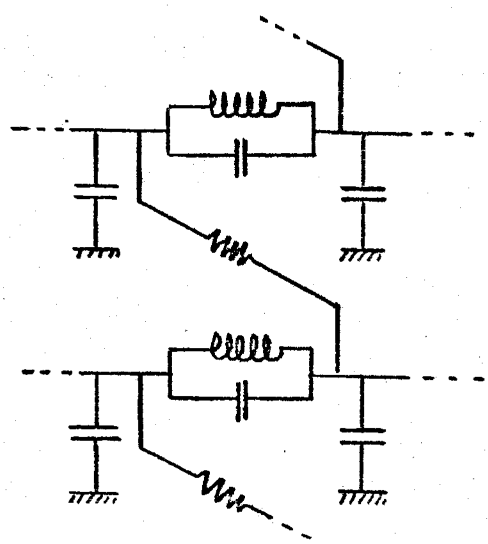


Fig. 1.1.4
First attempt at
an asymmetric filter

It was found that this did indeed have the desired effect although the particular characteristic obtained was not very useful because of a slope across the passband. However it showed that such networks were possible in principle and from this was developed the general theory and practice of sequence asymmetric polyphase networks. It was also subsequently found that such networks could be used for single sideband generation in a different mode to anything previously published and that a number of significant advantages made it worthy of further research and development.

1.2 Definition of Terms

Before proceeding to define the properties of sequence asymmetric filters it is useful to clarify the meaning of certain terms that will be used.

Polyphase Signals

A polyphase signal is a set of N voltages (or currents) of the same frequency. Such voltages may be represented as the projections of a set of N vectors onto some common axis in the plane of rotation.

In a symmetrical polyphase signal the vectors are equal in magnitude and spaced equally in phase.

Negative Frequency or Sequence

If for example a four-phase system is considered which has voltages of V, -jV, -V, +jV applied to its four input terminals then the input signal can be called symmetrical (because all voltages are equal in magnitude and spaced apart by steps of 90°)

and of say positive sequence (because, conventionally, all vectors rotate anticlockwise and the voltage on path 1 leads that on path 2 by 90°, similarly, path 2 leads on path 3 etc). If now the vectors rotate the opposite way the system is still symmetrical but is now of negative sequence (since 1 lags 2 by 90° instead of leading as before).

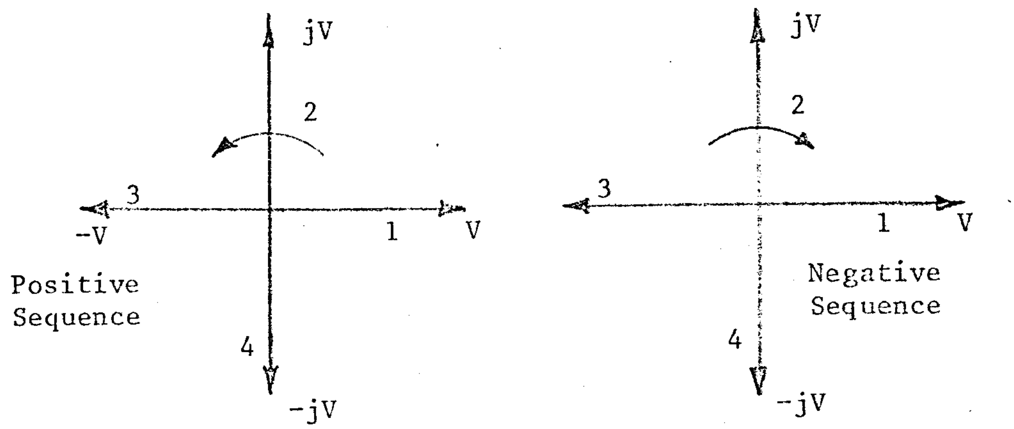


Fig. 1.2.1
Positive and Negative Sequence

If the voltage on path 1 is examined it will be seen to be V Sin ωt which is the projection of the vector (1) (rotating anticlockwise) onto the imaginary axis.

When the sequence of vectors is reversed, -V Sin ωt will be observed. Since -Sin ωt = Sin(-ωt) we can say that, on one phase, positive sequence represents positive ω and negative sequence represents negative ω.

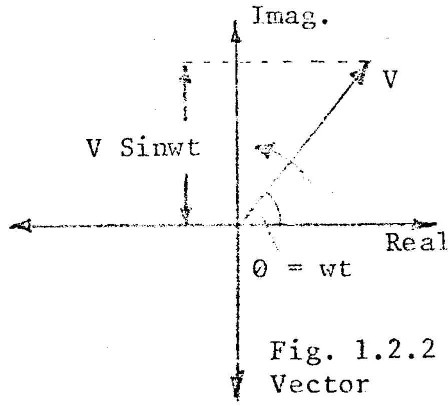


Fig. 1.2.2
Vector
Representation

Symmetrical Components

The theory of symmetrical components is a powerful tool in the analysis of unbalanced polyphase systems. According to this theory, any unbalanced system of N vectors can be represented as the sum of N symmetrical vector systems. Consider as an example Fig. 1.2.3 which shows the resolution of an unbalanced set of four vectors a_1', a_2', a_3' and a_4' into four symmetrical sets of four phase vectors. This can be expressed mathematically as

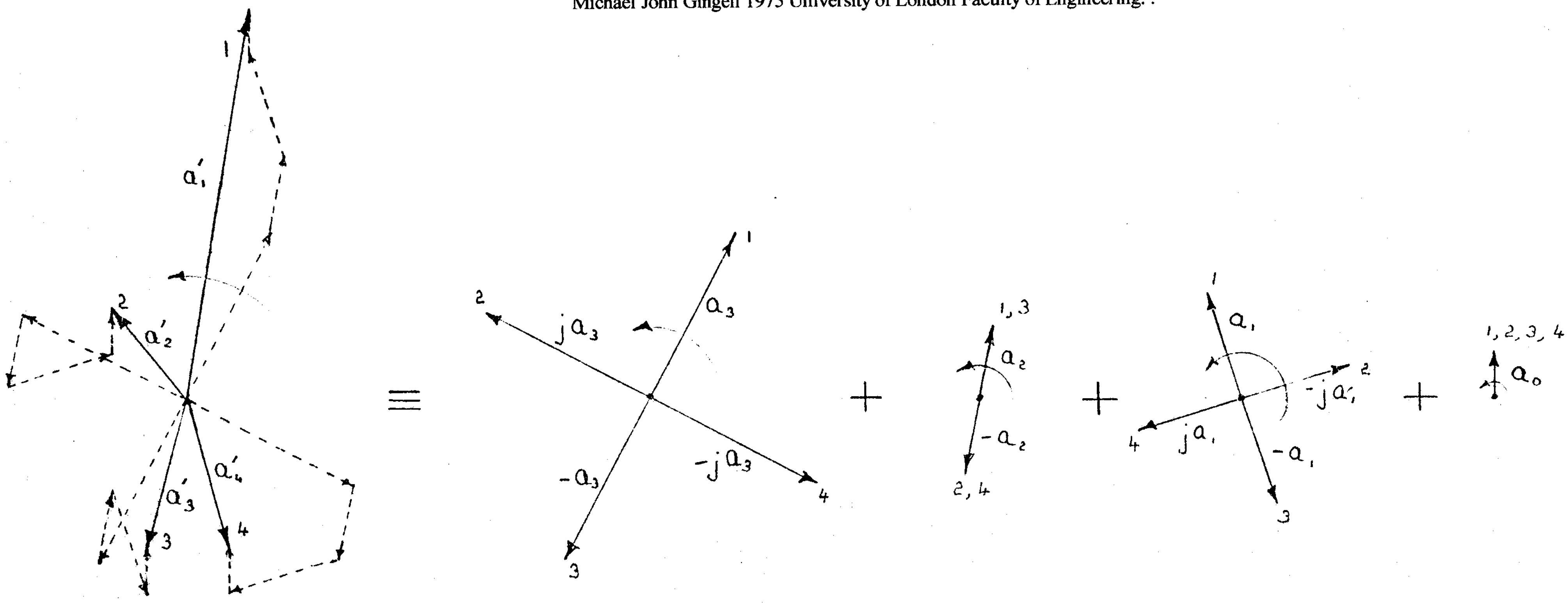
$$\begin{aligned}
a_1' &= a_o + a_1 + a_2 + a_3 \\
a_2' &= a_o - ja_1 - a_2 + ja_3 \\
a_3' &= a_o - a_1 + a_2 - a_3 \\
a_4' &= a_o + ja_1 - a_2 - ja_3
\end{aligned}$$

The symmetrical components can be derived by solving these equations which yields

$$\begin{aligned}
a_o &= \tfrac{1}{4}\left[a_1' + a_2' + a_3' + a_4'\right] \\
a_1 &= \tfrac{1}{4}\left[a_1' + ja_2' - a_3' - ja_4'\right] \\
a_2 &= \tfrac{1}{4}\left[a_1' - a_2' + a_3' - a_4'\right] \\
a_3 &= \tfrac{1}{4}\left[a_1' - ja_2' - a_3' + ja_4'\right]
\end{aligned}$$

Of the four symmetrical components (or sequences as they are known in power engineering) component 1 is positive in sequence since phase 1 leads phase 2 by 90^o. Similarly component 3 is of negative sequence. The sequences of components 0 and 2 are indeterminate so they are sometimes called zero sequence components (Ref. A4).

The general principle of analysis outlined can be extended to any number of phases and is fully dealt with in reference A4. Degenerate systems such as 2 phase quadrature systems can also be treated by the same method.



Unbalanced 4 phase signal = component 3 + component 2 + component 1 + component 0

Figure 1.2.3 Resulution of an asymmetrical signal into symmetrical components.

1.3 The Sequence Asymmetric Property

Consider the network shown in Fig. 1.3.1. The network is physically symmetrical in that each input node is connected to its respective output node via a transfer function A(p). Each input also connects to the output node above its next respective output via a transfer function B(p). In this context we consider real frequencies only ($p = j\omega$).

When, as shown, the networks is driven by a symmetrical polyphase input signal of negative sequence then the output will also be a symmetrical negative sequence signal but shifted in overall amplitude and phase according to the relationship:

$$\frac{VOUT}{VIN} = A(p) + jB(p) = H(p)$$

Positive sequence input signals will be affected differently:

$$\frac{VOUT}{VIN} = A(p) - jB(p) = H'(p)$$

Such a network can therefore be called "Sequence Asymmetric" since it exhibits different responses depending on the sequence of the polyphase input signal.

For analytical purposes the network of Fig. 1.3.1 can be replaced by that in Fig. 1.3.2 which has a complex transfer function

$$H(p) = A(p) + jB(p)$$

connected between each input node and its respective output node. This is correct for the negative sequence input

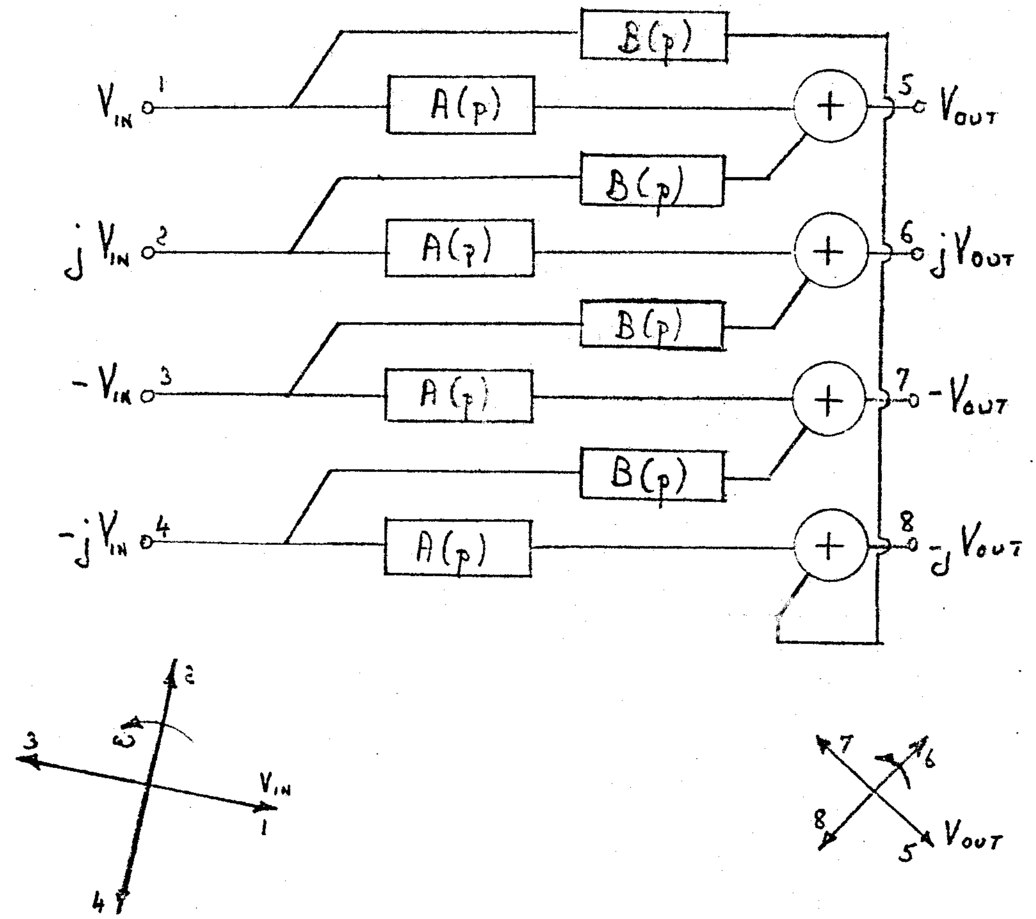


Figure 1.3.1 Sequence Asymmetric Polyphase Filter.

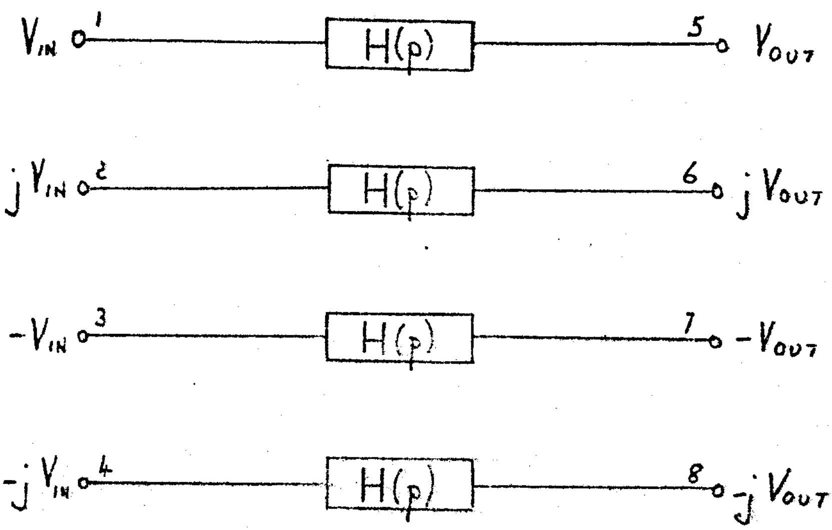


Figure 1.3.2 Complex Filter equivalent of Fig. 1.3.1

shown but for positive sequence inputs we would get

$$H'(p) = A(p) - jB(p)$$

Physically, the complex transfer function can be observed by driving the network from an N phase source and measuring the output on one phase. (See also on P.85).

Let A(p) and B(p) be ratios of polynomials in p with real coefficients such that

$$A(p) = A_1 + jA_2$$
$$B(p) = B_1 + jB_2$$

where A_1, A_2, B_1, B_2 are real and functions of ω.

$$\text{Thus} \quad H(p) = A_1 + jA_2 + j(B_1 + jB_2)$$
$$= A_1 - B_2 + j(A_2 + B_1)$$
$$\text{and} \quad H(-p) = A_1 + B_2 - j(A_2 - B_1)$$
$$\text{But} \quad H'(p) = A_1 + jA_2 - j(B_1 + jB_2)$$
$$= A_1 + B_2 + j(A_2 - B_1) = \overline{H(-p)}$$
$$\text{and} \quad H'(-p) = A_1 - jA_2 - j(B_1 - jB_2)$$
$$= A_1 - B_2 - j(A_2 + B_1) = \overline{H(p)}$$

Thus the response of the network for positive sequence inputs can be found by taking the network of Fig. 1.3.2 and substituting $\overline{H(-p)}$ for H(p). If the magnitude only is considered thus

$$|H'(p)| = |H(-p)|$$

It should be noted that although the argument has been pursued for the 4-phase case it can be proved for any other number of phases by generalising the operator j such that

$$H(p) = A(p) + e^{j\frac{2\pi}{N}} B(p)$$

where N is the number of phases.

The concept of reducing the complex network with real transfer functions to the simpler network of Fig. 1.3.2 with complex transfer functions is particularly useful in the synthesis procedure. Chapter 2 describes the single phase complex transfer function by various methods. Once the function H(p) is determined the polyphase equivalent can be generated either directly or by synthesising a single phase network which will contain imaginary elements. It will be shown in the Chapter 3 on lossless polyphase filters how these imaginary elements can be realised through interconnecting the phases.

1.4 Use in Single Sideband Modulation

It will be observed in later sections of this Thesis that the most economic circuits result from 4 phase designs. Consider therefore a 4 phase sequence asymmetric network designed to pass positive sequence signals over a frequency range from f_1 to f_2 and to stop negative sequence signals over the same frequency range as illustrated in Fig. 1.4.1. Let the network be driven from a single balanced input as shown in Fig. 1.4.2 where of the four input terminals, numbers 1 and 2 are connected together and 3 and 4 are connected together. By the theory of symmetrical components the input signal so connected can be resolved into the sum of two identical opposite sequence symmetrical components. This is shown in Fig. 1.4.3. The output of the network can be determined by considering its action on the two components separately and obtaining the overall result

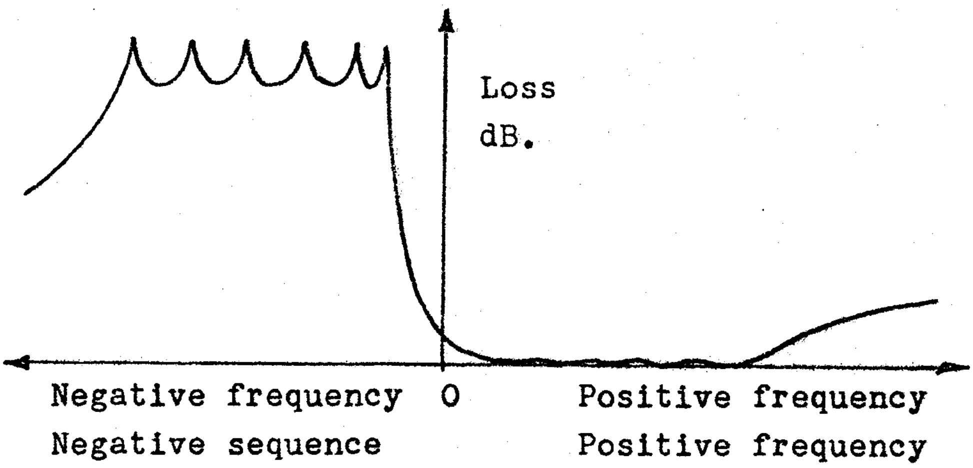


Figure 1.4.1 Frequency response of a network designed for single sideband modulation.

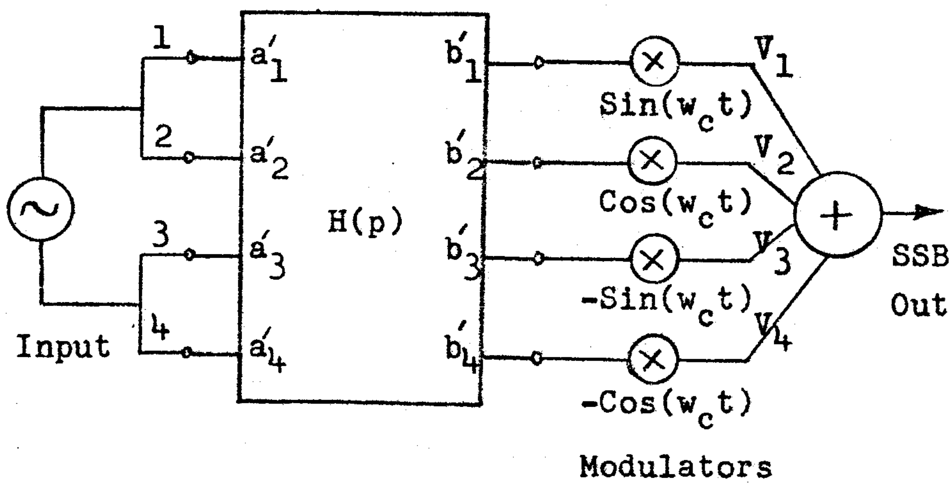


Figure 1.4.2 Single Sideband Modulator

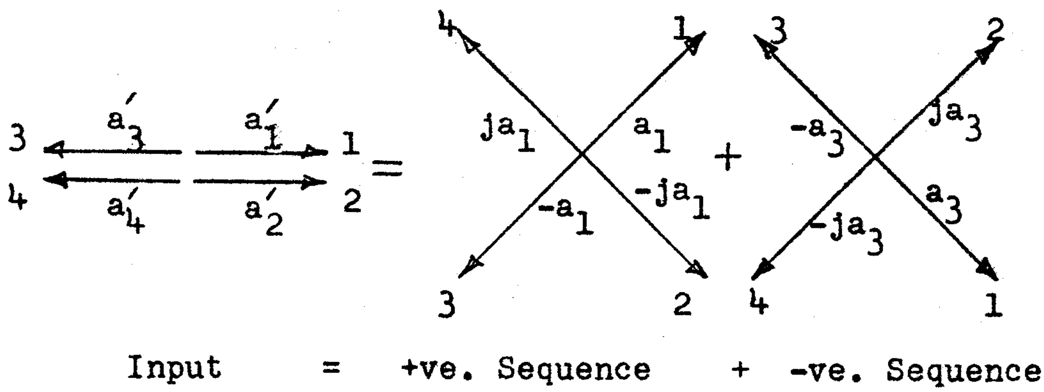


Figure 1.4.3 Input to filter of Fig. 1.4.2 split into symmetrical components.

by superposition.

The input signal to the 4 terminals is given by:

$$a_1' = V$$
$$a_2' = V$$
$$a_3' = -V$$
$$a_4' = -V$$

Resolving into symmetrical components gives

$$a_o = \tfrac{1}{4}\left[a_1' + a_2' + a_3' + a_4'\right] = 0$$
$$a_1 = \tfrac{1}{4}\left[a_1' + ja_2' - a_3' - ja_4'\right] = \tfrac{1}{2}(1 + j)V$$

positive sequence

$$a_2 = \tfrac{1}{4}\left[a_1' - a_2' + a_3' - a_4'\right] = 0$$
$$a_3 = \tfrac{1}{4}\left[a_1' - ja_2' - a_3' + ja_4'\right] = \tfrac{1}{2}(1 - j)V$$

negative sequence

The input is thus the sum of two opposite sequence components which will be filtered so that at the 4 output terminals we have by superposition

$$b_1' = b_1 + b_3 = a_1 H(p) + a_3 \overline{H(-p)}$$
$$b_2' = -jb_1 + jb_3 = j\left[-a_1 H(p) + a_3 \overline{H(-p)}\right]$$
$$b_3' = -b_1 - b_3 = -\left[a_1 H(p) + a_3 \overline{H(-p)}\right]$$
$$b_4' = jb_1 - jb_3 = -j\left[-a_1 H(p) + a_3 \overline{H(-p)}\right]$$

and after modulation

$$V_1 = V_3 = \left[\tfrac{1}{2}(1 + j) H(p) + \tfrac{1}{2}(1 - j) \overline{H(-p)}\right]\left[\frac{V(p-pc) - V(p+pc)}{2j}\right]$$

$$V_2 = V_4 = j \left[-\tfrac{1}{2}\ (1 + j)\ H(p) + \tfrac{1}{2}\ (1 - j)\ \overline{H(-p)} \right] \left[\frac{V(p-pc) + V(p+pc)}{2} \right]$$

where $p = j2\pi f$, f being the input and fc being the carrier frequency.

Summing the output of the 4 modulators gives

$$(1 - j)\ H(p)\ V\ (p-pc) + (1 + j)\ \overline{H(-p)}\ V\ (p + pc)$$

Note that this is mathematically identical to the output from quadrature modulation (ref. B7).

The first term represents the lower sideband attenuated by H(p) and the second represents the upper sideband attenuated by $\overline{H(-p)}$. The effect is therefore similar to conventional single sideband modulation where the signal is passed through a modulator followed by a sideband selection filter with a transfer function H(p-pc).

The action of the modulator circuit of Fig. 1.4.2 can be summarised in the following terms. The input signal is split into two components of positive and negative sequence. Each component is filtered and, when applied to the modulators, gives rise to only one sideband. By choosing the appropriate filter characteristic one sideband can be selected while the other is suppressed. The opposite sideband can be selected simply by reversing the modulator connections or the filter characteristic. The basic method can be used for any number of phases and with different network drive arrangements. The principle also works in reverse for demodulation.

The application of sequence asymmetric filters is discussed more fully in Chapter 5.